# Geometries → Models → Structures

Let no one
ignorant of
geometry
enter here

J. Baracs

# • A few words... •

One day recently, I decided to inventory the models displayed in my house. Besides the photos and the drawings, I added a short description for each. By doing this, I encountered a problem.

Why the models? Here, I show a photo of my office at the university before my retirement in 2000. It became well-known worldwide among the geometric and structural "mafia"; when they came on an academic visit to Montreal, they also visited my office. The models were student works accumulated during forty years of teaching and practicing spatial geometry. Once I retired, I found my house empty. So, I started constructing these models to satisfy my love of geometry, my interest in complex assemblies and my urge to create. As the photos suggest, I built many of them in my practice as a structural engineer.

I wrote these comments for friends who asked questions about the models. Those who did not ask questions probably feared the answer, and I could not blame them. Here lies my problem: each model has a geometrical-mathematical background, which was explained in my course on Structural Topology. Including all these detailed descriptions in this album would end up in a considerable volume, so I opted for a brief note addressed to those few who are familiar with the subject. My apologies…

A final note: I used the software Geometria 2° to produce this album. My friend Louis Langlois and I developed the software in the last few years.

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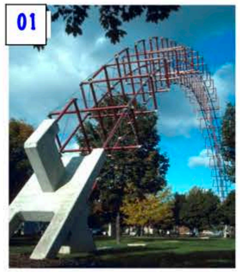
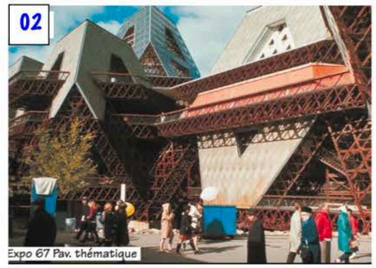
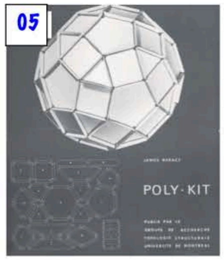
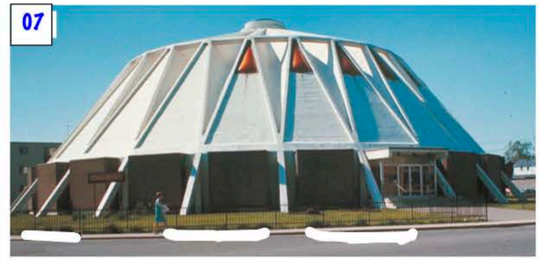


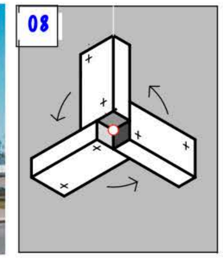



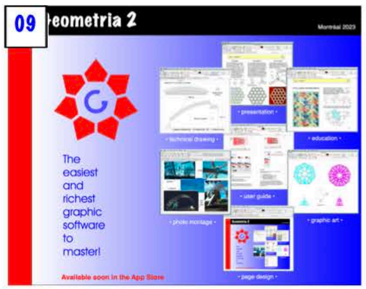


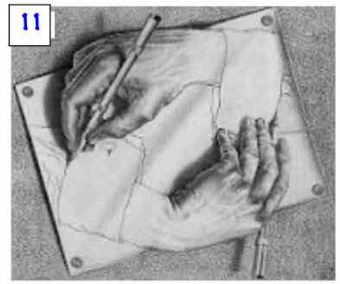




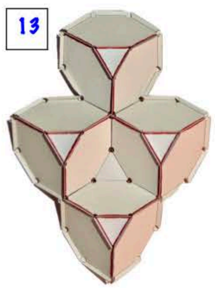



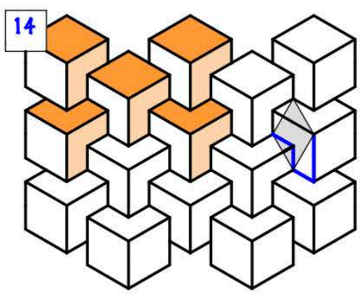





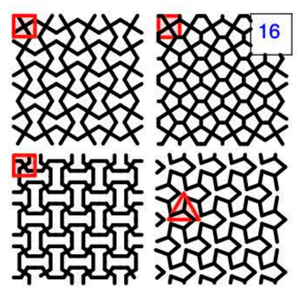


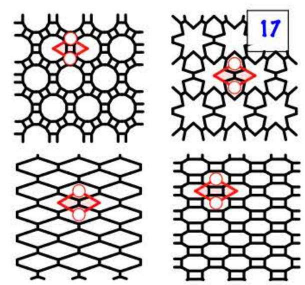




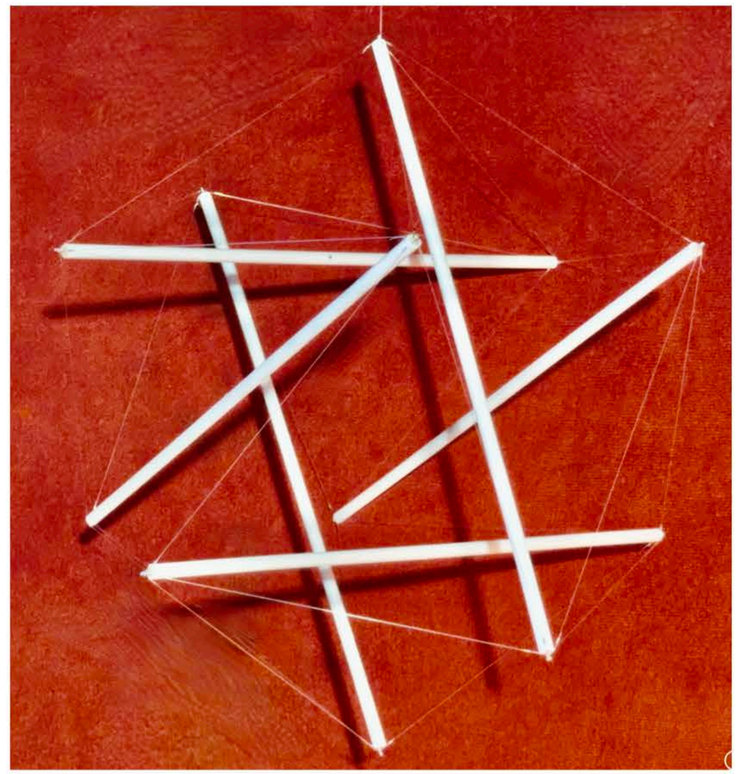


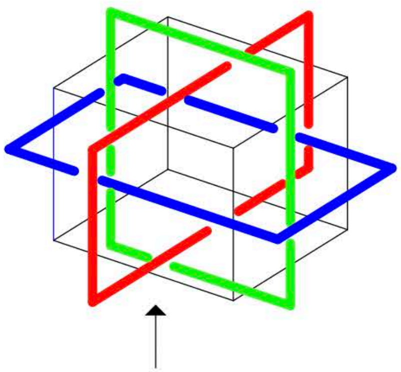


# • Tensegrity •

The two photos on the right show a monument called L'espace Mascoutain, built in St Hyacinthe, Québec. The structure is composed of 24 prefabricated tensegrity units, spanning 80 m with a height of 16.5 m.
The unit contains 3 rectangles of rigid members and 24 tension members. The rectangles do not touch each other.
Buckminster Fuller patented the tensegrity structures in 1964, but they were known well before that. Kenneth Snelson, an American sculptor, introduced them much earlier in the USA.
The photo of the model shows the basic unit, following the geometry of the icosahedron.

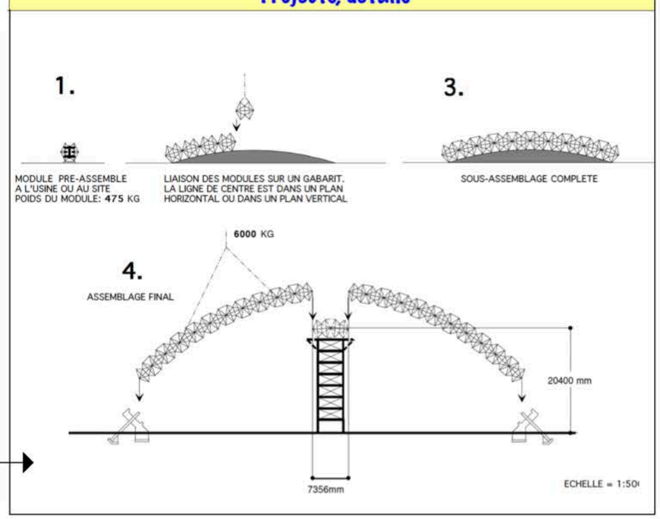




The blue, green and the red rectangles are the compression members.

The assembly of the structure depicts the 20 m high temporary support.

Projects, details

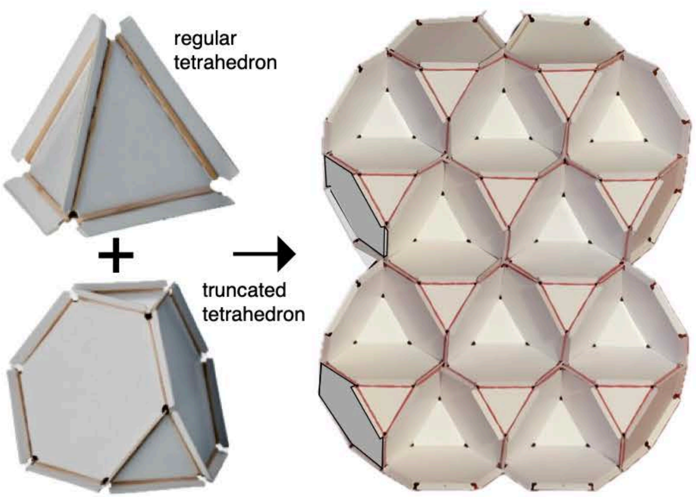


# • Expo 67 Montreal •

At Expo 67, the Canadian pavilions were based on a geometry not used before. The buildings were composed of two polyhedra: the tetrahedron and the truncated tetrahedron. These two polyhedra fill the space without gaps or overlaps. Following this geometry, floors, walls, and supports were constructed, which resulted in inclined walls and supports. At the time, this integration of building components was also an innovative approach, widely published in the international media.

The model of a set of truncated octahedron shown on the right is made of five mm thick polystyrene plates, hinged together with eyelets and bolts.

The white elongated hexagons represent the orthogonal projection of the structure on the wall.



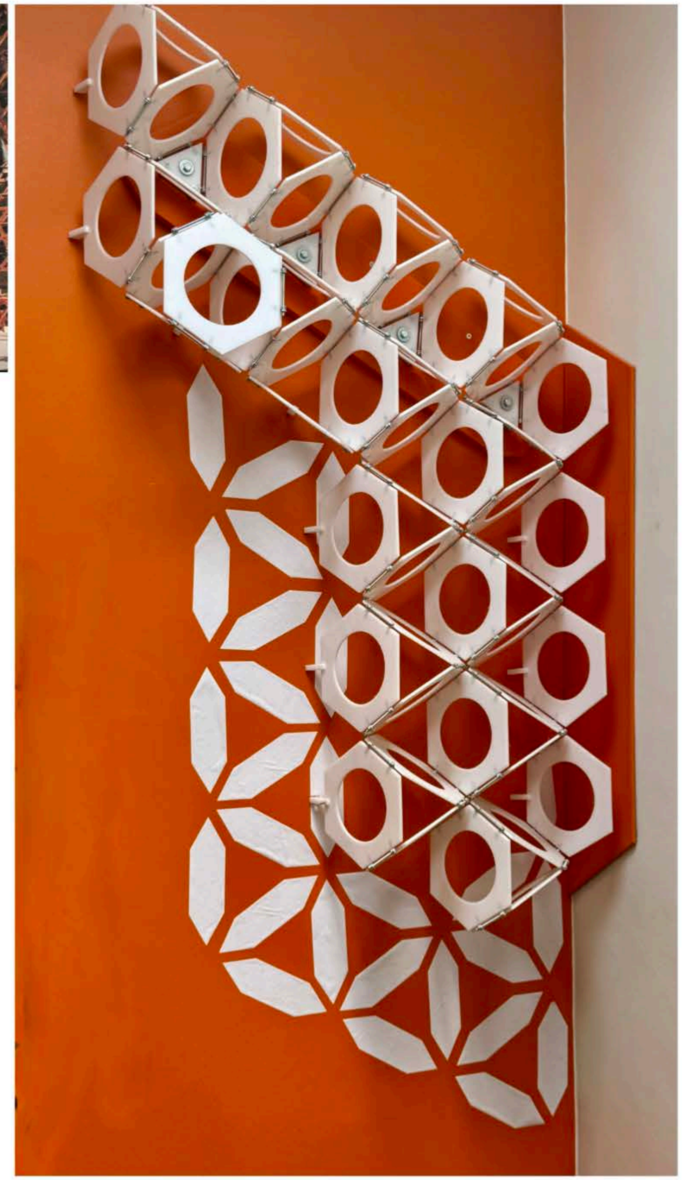


**3.04 The 5 regular polyhedra**

| code | name | symbol | faces | v | e | f |
|---|---|---|---|---|---|---|
| R 01 | tetrahedron | 3.3.3 | 4 A | 4 | 6 | 4 |
| R 02 | octahedron | 3.3.3.3 | 8 A | 6 | 12 | 8 |
| R 03 | cube | 4.4.4. | 6B | 8 | 12 | 6 |
| R 04 | icosahedron | 3.3.3.3.3 | 20A | 12 | 30 | 20 |
| R 05 | dodecahedron | 5.5.5 | 12 C | 20 | 30 | 12 |

**3.05 The 13 + 2 semi-regular polyhedra**

| code | name | symbol | faces | v | e | f |
|---|---|---|---|---|---|---|
| S 01 | truncated tetrahedron | 3.6.6 | 4A+4D | 12 | 18 | 8 |
| S 02 | truncated cube | 3.8.8 | 8A+24E | 24 | 36 | 14 |
| S 03 | truncated octahedron | 4.6.6 | 6B+8D | 24 | 36 | 14 |
| S 04 | Cub-octahedron | 3.4.3.4 | 8A+6B | 12 | 24 | 14 |
| S 05 | truncated cuboctahedron | 4.6.8 | 12B+8D+24E | 48 | 72 | 26 |
| S 06 | Rhombi-cuboctahedron | 3.4.4.4 | 8A+18B | 24 | 48 | 26 |
| S 07 | snub-cube | 3.3.3.3.4 | 32A+6B | 24 | 60 | 38 |
| S 08 | truncated dodecahedron | 3.10.10 | 20A+60F | 60 | 90 | 32 |
| S 09 | truncated icosahedron | 5.6.6 | 12C+20D | 60 | 90 | 32 |
| S 10 | icosi-dodecahedron | 3.5.3.5 | 20+12C | 30 | 60 | 32 |
| S 11 | truncated icosidodecahedron | 4.6.10 | 30B+20D+60F | 120 | 180 | 62 |
| S 12 | rhomb-icosidodecahedron | 3.4.5.4 | 20A+30B+12C | 60 | 120 | 62 |
| S 13 | snub-dodecahedron | 3.3.3.3.5 | 80A+12C | 60 | 150 | 92 |
| S 14 | n-gonal-prisms | n.4.4 | nB+2n | 2n | 3n | n+2 |
| S 15 | n-gona-antiprisms | n.3.3.3 | 2nA+2n | 2n | n | 2n+2 |

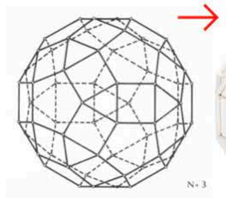


# Geometries → Models → Structures 3.

# • A polyhedral lamp •

**Following the Poly-Kit system, this lamp is built with 72 hand-cut polystyrene polygons, the edges attached with rubber bands. Using LED bulbs, eliminate fire hazards since no heat is generated. The truncation of the 5-valent vertices, where 5 edges meet, results in a visually pleasing network.**

Projection of the semi-regular polyhedron



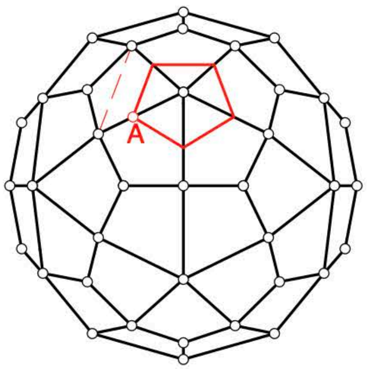
Model of the semi-regular polyhedron

Dual of the semi-regular polyhedron

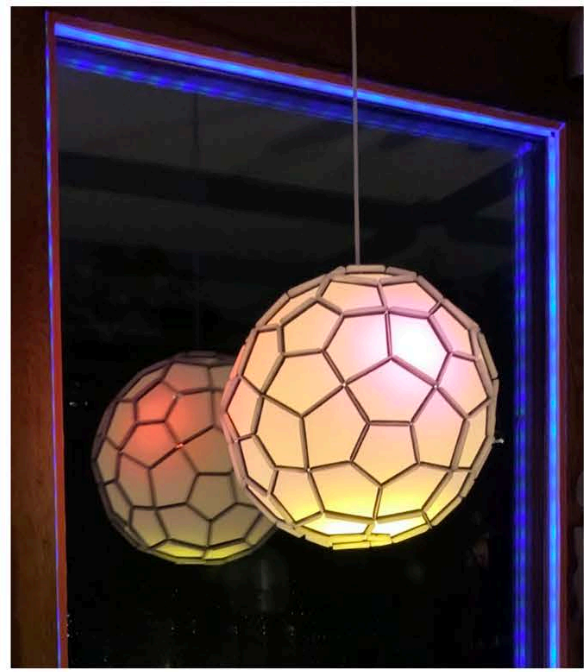
Semi-regular polyhedra

**3..07 The 13 + 2 dual semi-regular polyhedra**

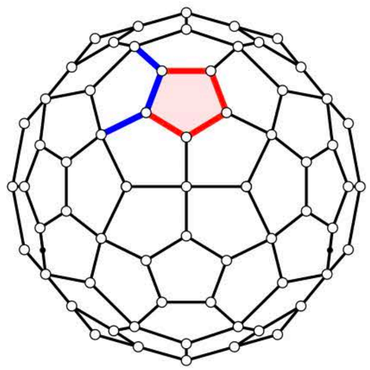
| code | name | symbol | faces | f | e | v |
|---|---|---|---|---|---|---|
| D 01 | triakis tetrahedron | 3.6.6 | - | 12 | 18 | 8 |
| D 02 | triakis octahedron | 3.8.8 | - | 24 | 36 | 14 |
| D 03 | tetrakis hexahedron | 4.6.6 | - | 24 | 36 | 14 |
| D 04 | rhombic dodecahedron | 3.4.3.4 | 12G | 12 | 24 | 14 |
| D 05 | hexakis octahedron | 4.6.8 | - | 48 | 72 | 26 |
| D 06 | trapezoidal icositetrahedron | 3.4.4.4 | - | 24 | 48 | 26 |
| D 07 | triakis icositetrahedron | 3.3.3.3.4 | - | 24 | 60 | 38 |
| D 08 | triakis icosahedron | 3.10.10 | - | 60 | 90 | 32 |
| D 09 | pentakis dodecahedron | 5.6.6 | - | 60 | 90 | 32 |
| D 10 | rhombic triacontahedron | 3.5.3.5 | 30H | 30 | 60 | 32 |
| D 11 | hexakis icosahedron | 4.6.10 | - | 120 | 180 | 62 |
| D 12 | trapezoidal hexacontahedron | 3.4.5.4 | - | 60 | 120 | 62 |
| D 13 | pentagonal hexacontahedron | 3.3.3.3.5 | - | 60 | 150 | 92 |
| D 14 | n-gonal dipyramids | n.4.4 | - | 2n | 3n | n+2 |
| D 15 | n-gonal trapezohedra | n.3.3.3 | - | 2n | n | 2n+2 |




This polyhedron, the trapezoidal hexecontahedron, is the well-known dual of the semi-regular polyhedron called the rhombicosidodecahedron.
It has 60 identical trapezoid (deltoid) faces, 120 edges and 62 vertices. All edges are tangent to a sphere (intersphere), and all faces are tangent to a smaller sphere (inscribed sphere).
It has 12 5-valent vertices, following the five-fold symmetries of the regular dodecahedron and two different edge lengths.



I truncated the 12 5-valent vertices of the dual polyhedron, as shown on the right. This is not a discovery or an invention but rather a pleasant surprise.
The sphere is now covered with 72 pentagons, 12 of them are regular, and the other 60 are symmetrical. My conjecture: 72 is the maximum number of pentagons (non-regular) that can cover the sphere.
While the choice of the point A is arbitrary, it can satisfy three criteria: 1. the new edges are tangent to the intersphere, 2. the new face tangent to the inscribed sphere, 3. the blue edges are of the same lengths (how to construct A?).



# Geometries → Models → Structures 4.

## • The octet truss •

Octet is an abbreviated name for a space frame composed of regular octahedra and tetrahedra. Space frames are three-dimensional structures: bars are joined in nodes, forming a geometrical pattern. Alexander Graham Bell is credited with the invention of the octet truss. It is used in engineering practice to span large spaces due to its efficiency: bending moments are absorbed only by tension or compression forces in the bars.
Since tetrahedra and octahedra fill the space without gaps or overlaps, it is possible to extract two different slices: a.) square gride (half octahedra) and b.) triangular gride. The square grid structure is not stable in itself; it requires additional restraint.

regular octahedron

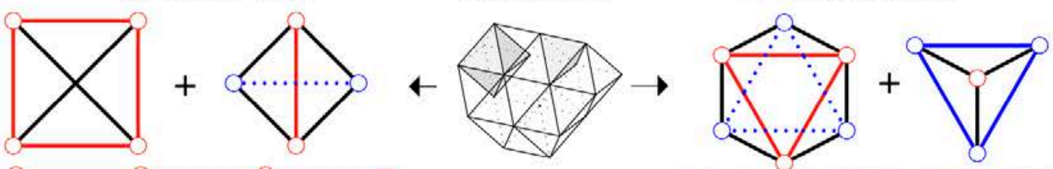
regular tetrahedron

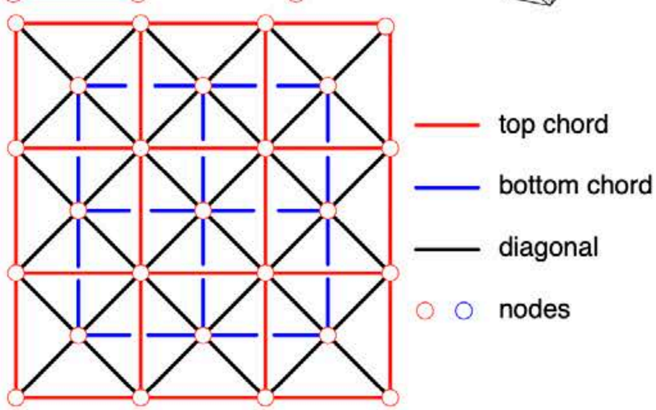
The first reinforced concrete space frame, CDN Plaza, Montreal

Wooden octet truss, triangular grid, plastic nodes, student project

Octet truss, sqare gride, nodes and bars made of PVC pipes

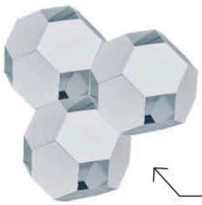
a.) Square gride | Juxtaposition | b) Triangular gride



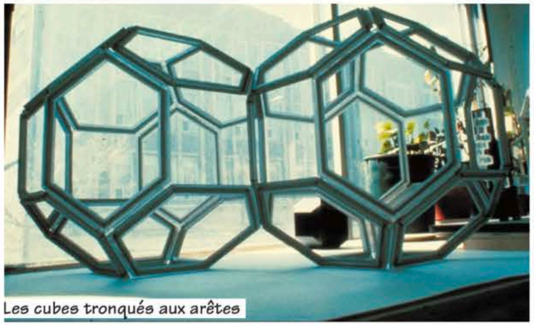



Metro Côte des Neiges

# • The edge truncated cube •

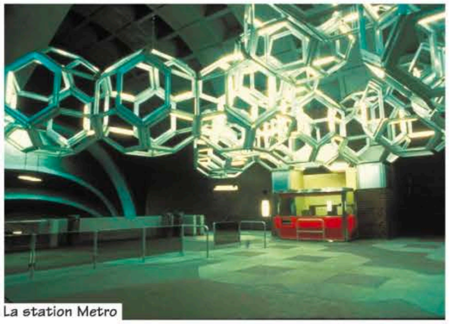


If you truncate the 12 edges of a cube in a symmetrical, repetitive manner, the result is the polyhedron is shown below; it has no particular name. The edges of the cube are replaced by non-regular but symmetrical, 12 identical hexagonal faces, while the square faces of the cube are truncated into 6 smaller square faces.
This little-known polyhedron is also remarkable because it fills in space if combined with cubes.



Les cubes tronqués aux arêtes

Models of the edge truncated cubes, prepared for the Metro project



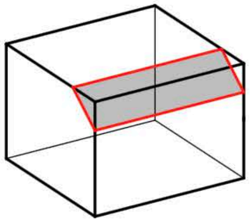
La station Metro

The suspended space-structure of the Namur Metro station in Montréal

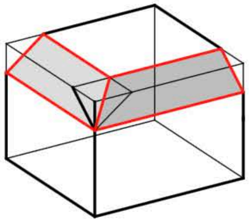

Les panneaux articulés

Innovative kiosks built at the Complex Desjardin, in Montréal to celebrate the 30th anniversary of the Faculté de l'aménagement, Université de Mtl. This is the first application of the new concept of "Articulated Spatial Panel" structures.

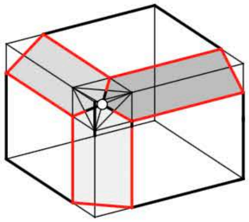
One edge is truncated, the secant plane is at 45° to the cube's face

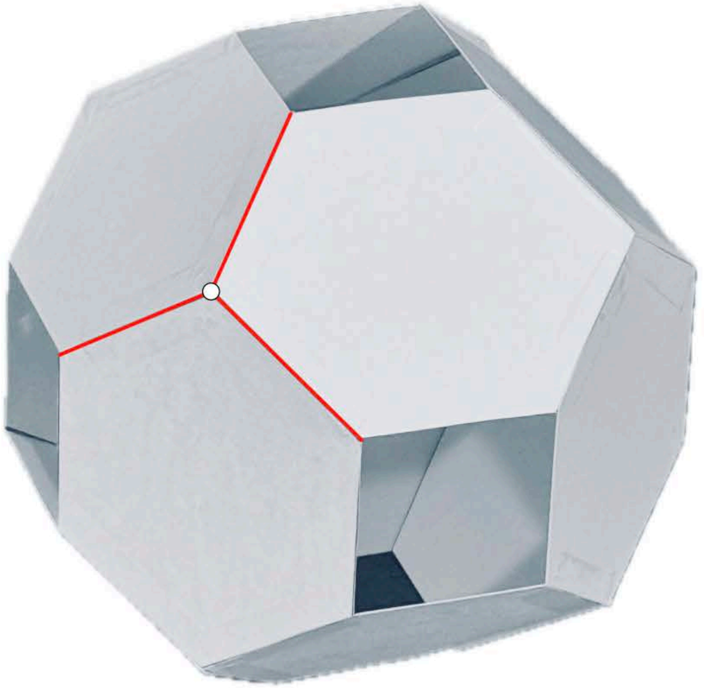


Two adjacent edges are truncated, the two secant planes meet in a line



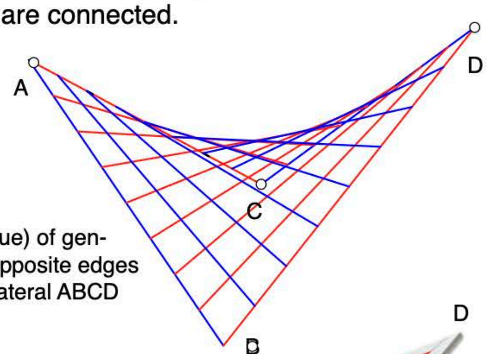
Three adjacent edges are truncated, the three secant planes meet in a point



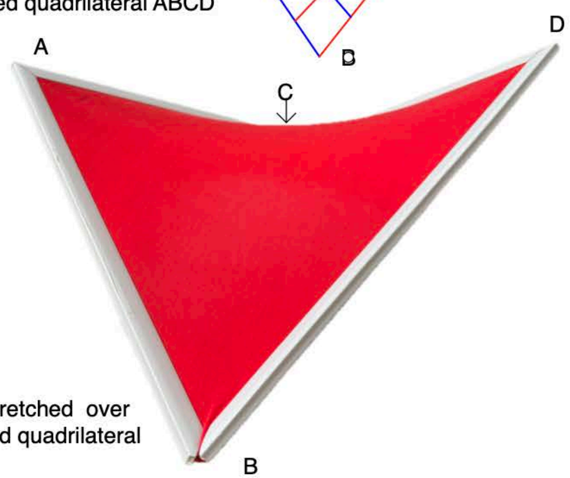


# • Hyperbolic paraboloides •

One of the most fascinating surfaces defined by a simple mathematical function is the hyperbolic paraboloid. It is a double-curved surface also called saddle-shape because of the two principal curvatures one is convex, and the other is concave.
The drawing below shows an attractive property of the surface: despite the apparent curvatures, the surface is constructed by two sets of straight lines called generators. The two non-coplanar segments, AB and BC, are divided into the same number of parts and the corresponding points are connected.




two sets (red and blue) of generators are linking opposite edges of a skewed quadrilateral ABCD



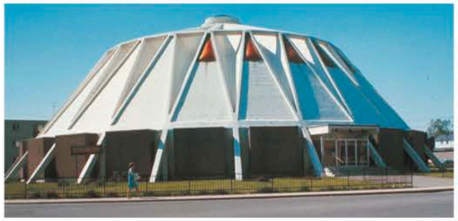

fabric is stretched over the skewed quadrilateral ABCD

**1. St Jean-Brebeuf church, Ville Lasale QC.** The structure is composed of 24 identical prefabricated concrete HP units.

**2. Place de Quebec in Paris.** The stone squares of the base form an HP surface.

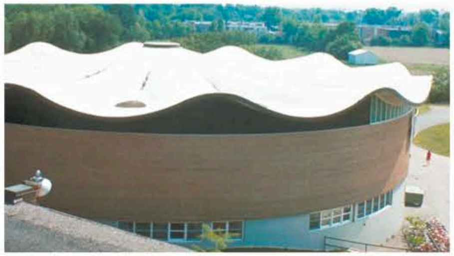
**3. Riverdale High School in Pierrefonds, QC**. The roof is covered with 12 identical concrete HP units.

1.

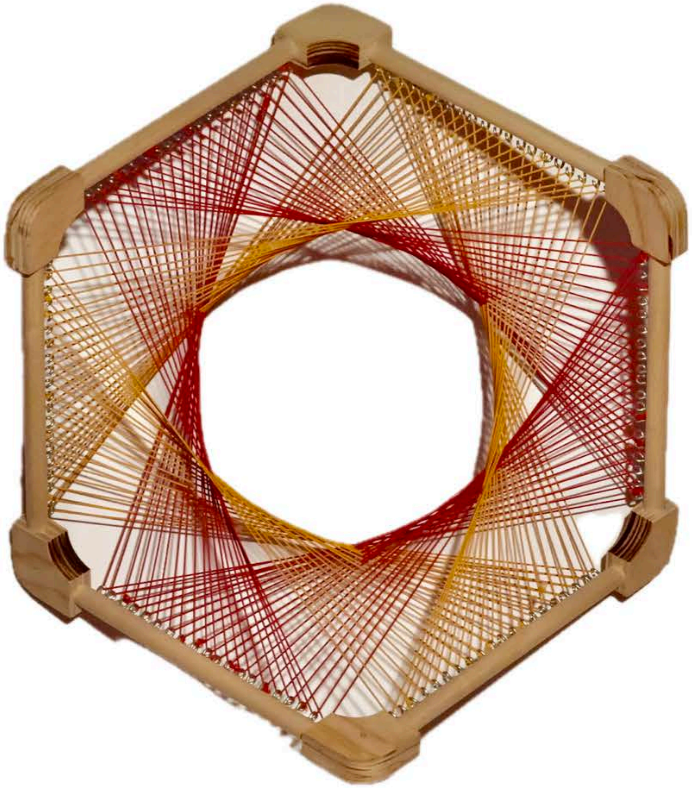


2.

3.



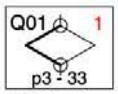
If you remove two diagonally opposite vertices of a (regular) cube with the six incident edges, you end up with a skew hexagon with six contiguous edges. Each facing two non-coplanar edges. Thus, two HP surfaces are issued from every edge by one set of generators, heading to the two skew ledges. Since every HP surface is counted twice; we have altogether six HP surfaces intersecting in an orderly manner.
The model required a rigid 90° joint between the rods.



# • A new joint •

## the pattern

Here, we present an example of viewing a 2-dimensional drawing of a pattern as a projection of a 3-D model. Create the blue motif inside the half-tile Q01 (Fig.1) and rotate it 3-fold around the center of the triangle (Fig. 2). The new motif is now 2-fold rotated around the center A, and we obtain the final motif in Fig. 3. Rotate 3-fold the final motif around the center B, and the pattern is obtained.




3-fold rotation 2-fold rotation 3-fold rotations

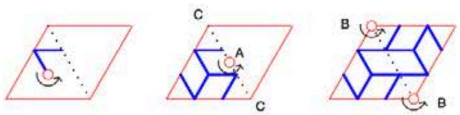
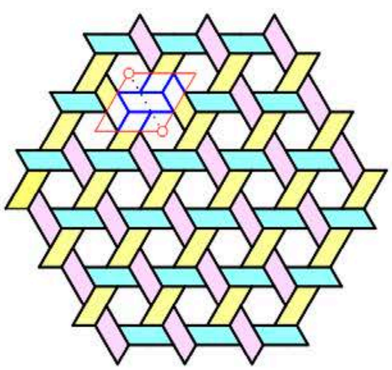
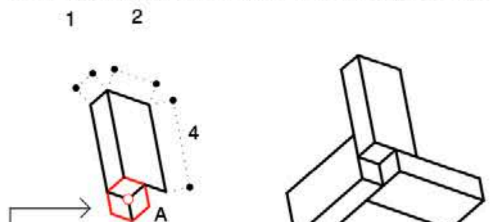



Fig. 1 Fig. 2 Fig. 3

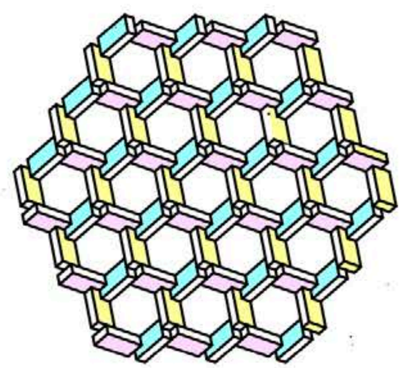


## the projection

Any 2-D drawing may be viewed as a central, parallel or orthogonal projection of a 3-D configuration and can be lifted into 3-D. Looking at the pattern on the left, the colouring helps to recognize how the parallelograms (or the rectangles) are joined together in 3-D. To build a solid model, we added thickness to the plane figures. It was simple to prepare the orthogonal projection: the three identical rectangular prisms meet in a new type of joint at the common point A.

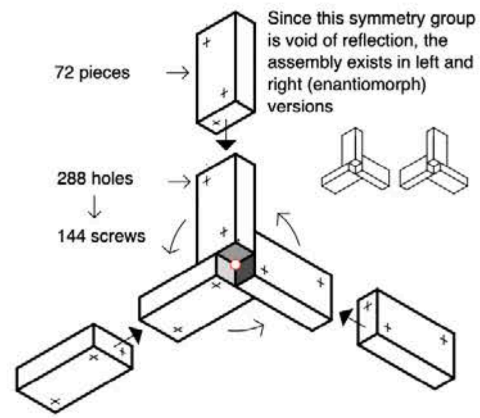


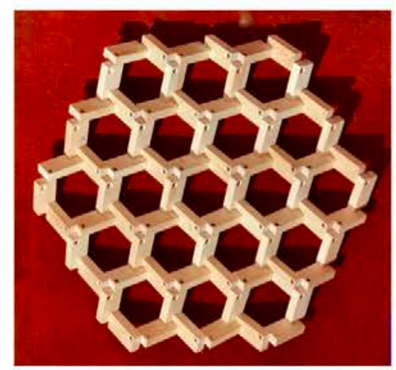

The red regular hexagon is the projection of a skew hexagon



## the 3-D model

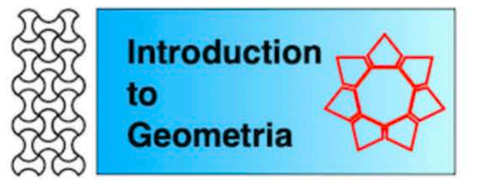
This joint is the product of a 3-D symmetry operation: a rectangular prism is rotated 3-fold around the "vertical" body-diagonal of the virtual cube defined by the three shaded faces.



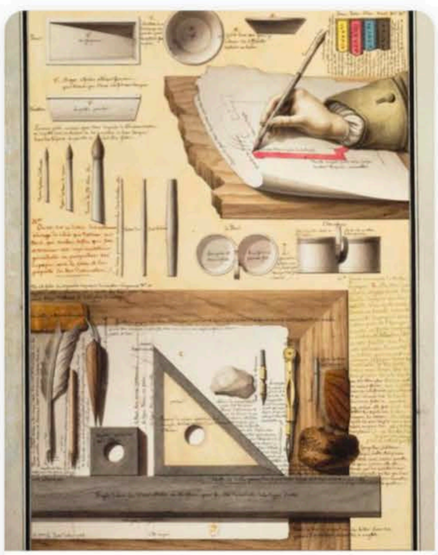
# • Geometria 2 •

Geometria Help

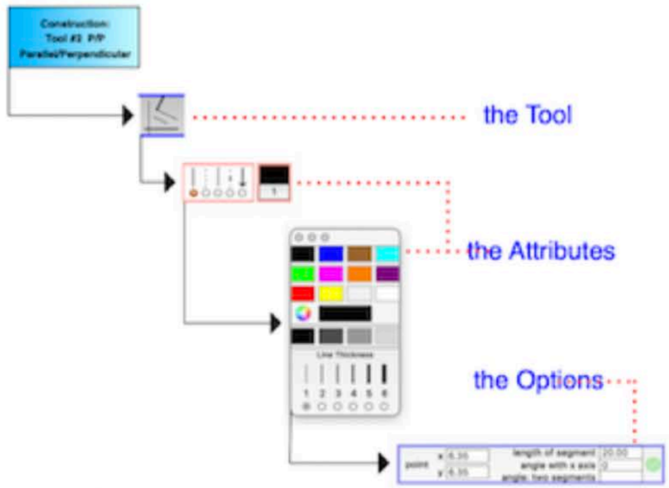


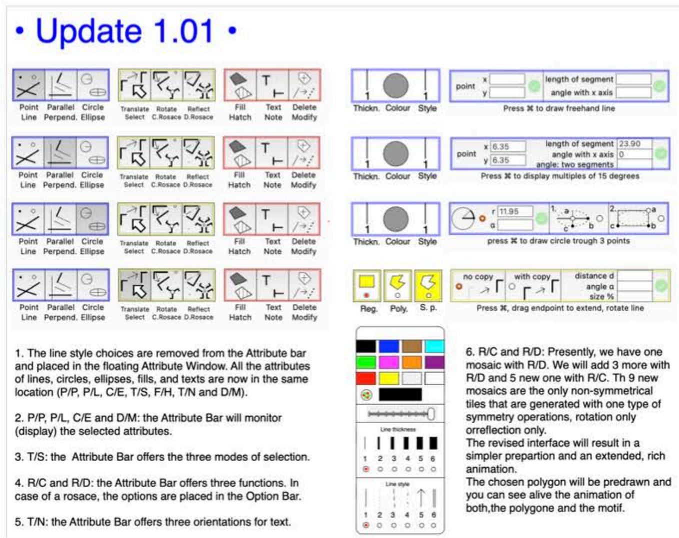

## A very short history of drafting.

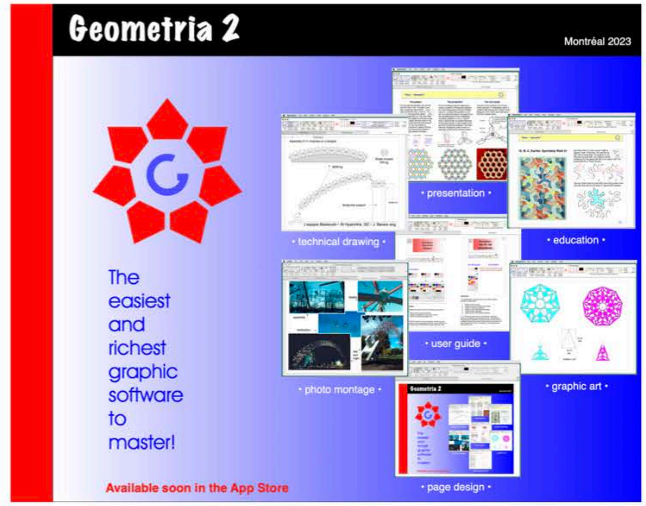
The essential drafting instruments, the ruler, the compass and the protractor, have been with us for over two thousand years with little change. The tableau below by the 18. century architect J. Le Queu illustrates the inventory of traditional drafting tools:



## the interface

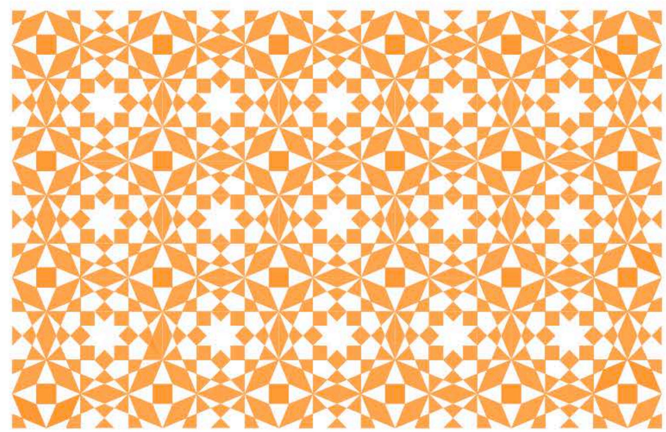



## the WEB page

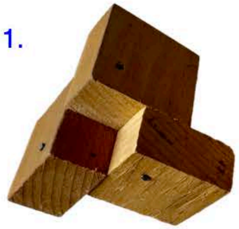
## pattern created with Geometria©



# • Cubes •

Here, we present cubes or cubic joints in different contexts. The new joint in Fig. 1 is joined by screws, the joint in Fig.2 is held by nails. Surprisingly, the latter is just as strong as the former: the three nails are perpendicular to each other, thus pulling apart the joint is difficult. This joint is used to build the models in Figs 6 and 7. Please note that the centerlines of the edges (rods) are meeting in a point, following an accepted structural and geometric criteria.

1.



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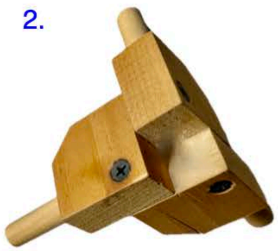

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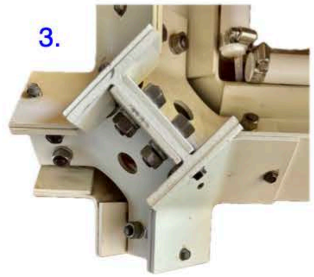
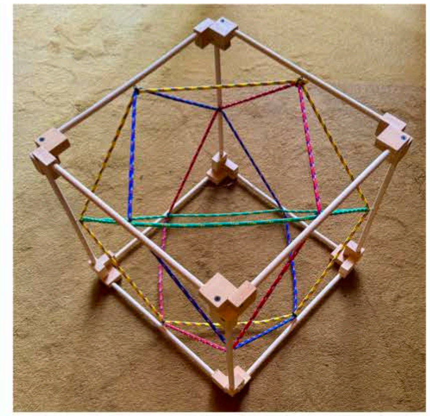


This is a metal joint for a cubic structure built in the Palais de justice in Quebec City. The four horizontal, and the two vertical “H” sections all perpendicular to each other, are bolted together with gusset plates.

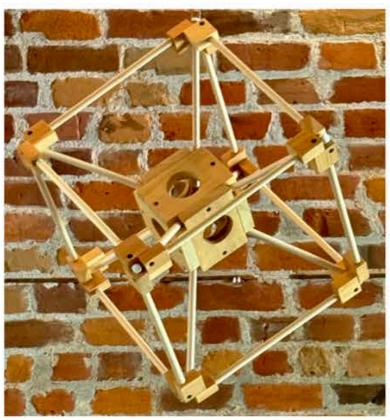
4. This complex surface is stretched over a skewed hexagon. The six bars are edges of a (regular) cube. See the skewed hexagon on P. 5



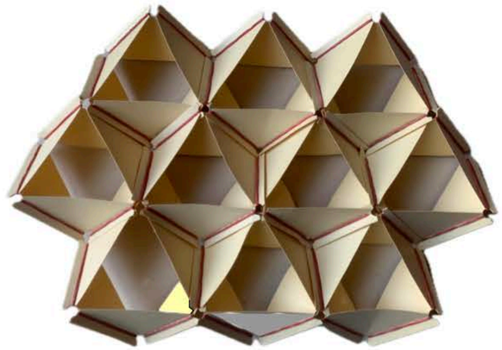
5. An interesting property of the cube: when joining the mid-points of the proper edges, we receive four regular hexagons.

6. Compare this model with the skew hexagon on P. 5. Here, we also have six HP surfaces, but only 1 per edge.

7. This cubic configuration is the 3-D projection of the 4-D cube called the hypercube. It has 16 vertices, 32 edges …

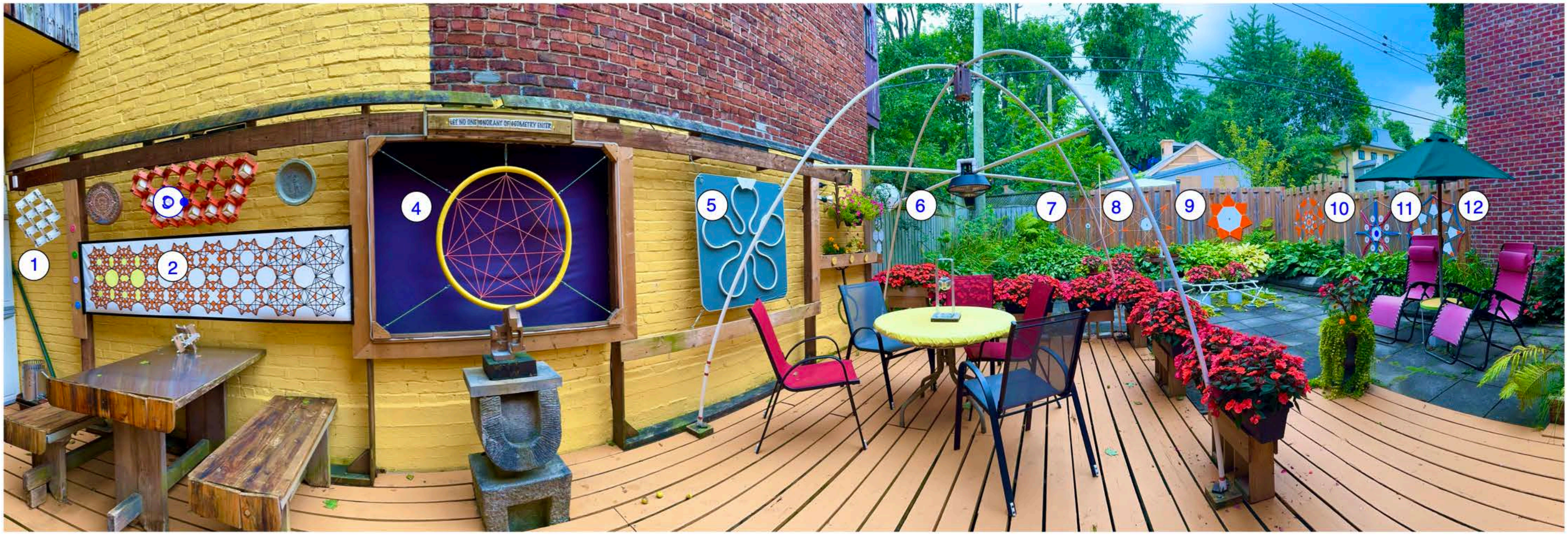


8. The cubes are not easy to detect here. The cubes are truncated by two parallel planes in equilateral triangles and juxtaposed. The truncated cubes fill the space with non-regular but symmetrical tetrahedra (see Octet Truss)



9. Here, thin rectangular prisms are assembled in a cubic structure, also based on the joint in Fig.1.The face plates are identical.

# • The garden at 21 •




**1.** 30mmx30mm polystyrene angles are attached pairwise, resulting in "Z" sections. These are attached by bolts to form a hexagonal (!) grid.

**2.** The pattern is created with the software Geometria: 16 dihedral rosacea are juxtaposed. Each rosace has eight mirrors and an 8-fold rotational center.

**3.** Same idea as the model in "2-D to 3-D", similar to n° 1. 24mmx38mmx77mm wooden blocks are bolted together to form a hexagonal pattern.

**4.** A complete graph of seven points: every vertex is connected to every other vertex, with 21 edges. See variations on regular heptagons (7., 8. and 9.).

**5.** This is a light tube bent into the shape of a dihedral rosace. The symmetry is 5-fold. The motif is composed of two circular arcs.

**6.** Trapezoidal icositetrahedron, dual of the semi-regular rhombicuboctahedron. The twenty-four faces are identical circles, inscribed in the dual faces.

**7.** A regular, concave heptagon. Every second vertex is connected with an edge in contiguity. All edges and angles are of equal size.

**8.** A regular, concave heptagon. Every third vertex is connected with an edge in contiguity. The heptagon is the first polygon with two distinct concave varieties.

**9.** This is a design based on the regular heptagon. We use it as a logo for our software Geometria. The material is 3mm polyester sheet.

**10.** This is the "decorated" 2-D graph of the 4-dimensional cube called a hypercube. The 3-D projection of the hypercube is the model shown in "Cubes"

**11.** An exercise in graphic design, using elements of the 2-D graph of the 4-D cube. The materials used are ropes and plumbing pipes.

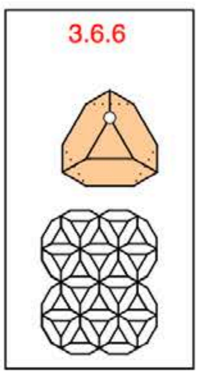
**12.** Similar to 11., we took advantage of the flexibility of the the small-diameter plumbing pipes.

# • Polyhedral surfaces •

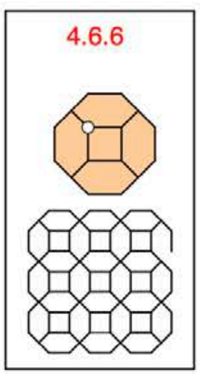
This idea came accidentally during the preparation of this new version of Poly-Kit. I attended a conference on ballet in a public hall at Place des Arts in Montreal. I looked at the acoustic ceiling recently installed, and I cringed. It was a visual disarray, an insult to 3-D geometry. Next day engrossed in the possibilities, I started to think, draw, and build models, each step bringing me closer to a better understanding of this new concept: regular polyhedral surfaces (RPS).

These surfaces are extracted as layers from juxtaposed regular and semi-regular polyhedra. In the three cases below, we indicated the incidence symbol of the principal semi-regular polyhedra. The models of the variations are depicted on the right. The model 1a. It is included to show that the polygons may not need to be regular: the hexagons are not regular but symmetrical. Model 2. is a light fixture at 21 Springfield.

1.




2.

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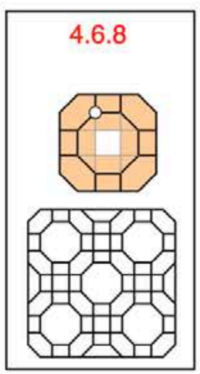
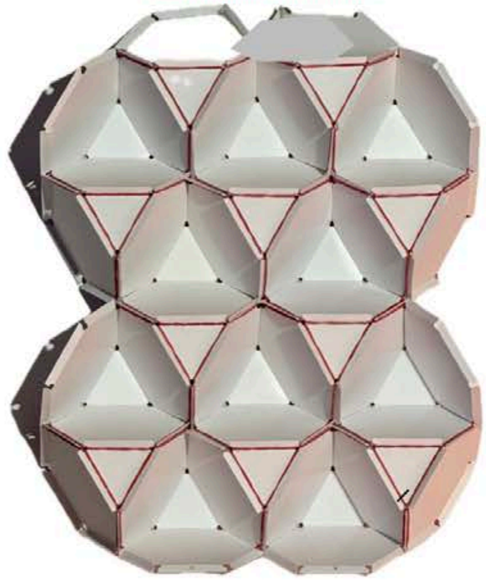
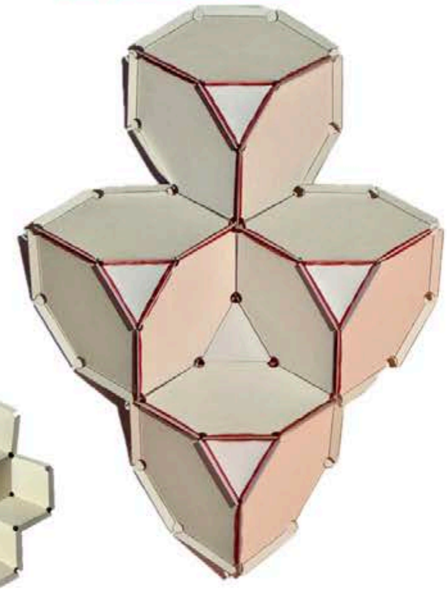



1. 3.6.6 truncated tetrahedron

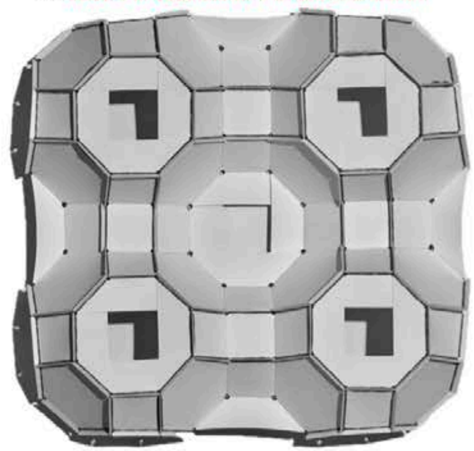


1a. 3.6.6 truncated tetrahedron



2. truncated octehedrron

3. cube + tr. octahedron + tr. cuboctahedron

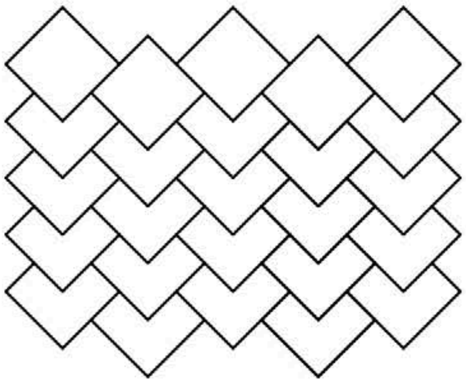


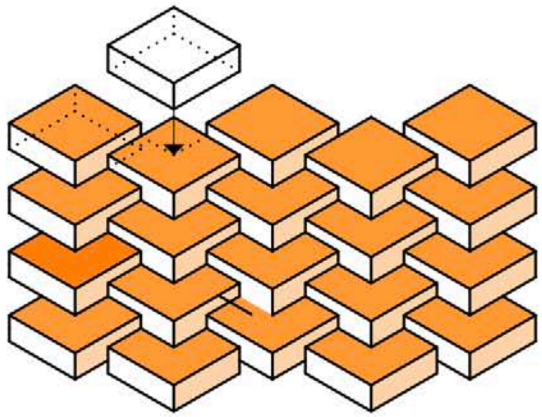
# • Polyscala - Multistair •

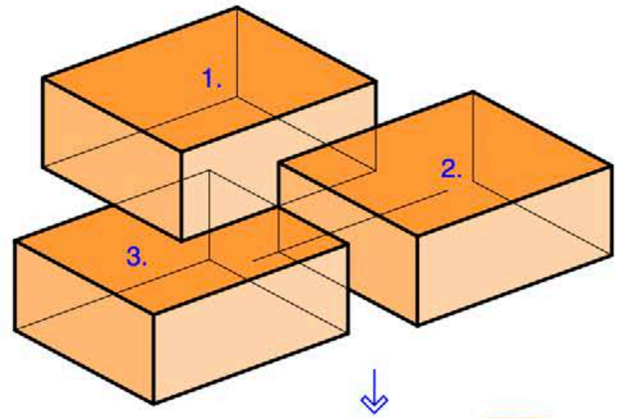
I had some difficulty with naming this assembly. Using some ratios, it looks like a stair with a great number of options to descend from the top to bottom.

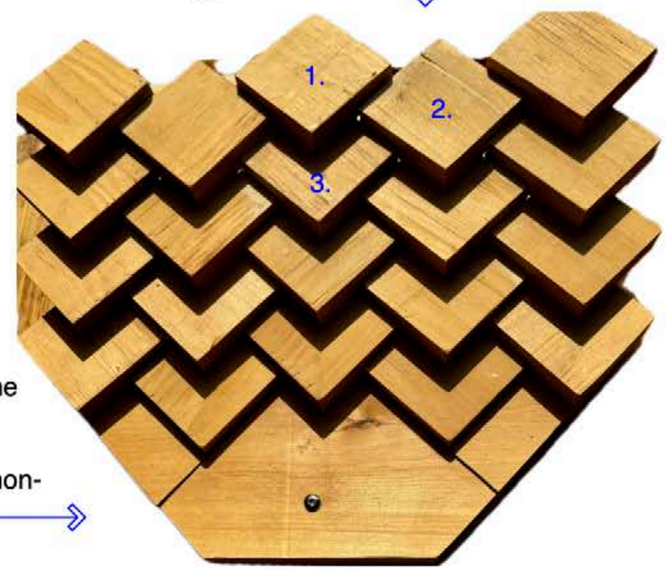
The drawing below shows the assembly of identical square-based prisms. If cubes are used instead of the thin prisms, the three cubes shown below present a three-fold rotational symmetry.

### From 2-D to 3-D

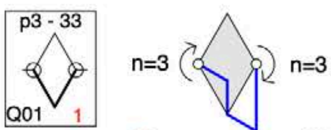




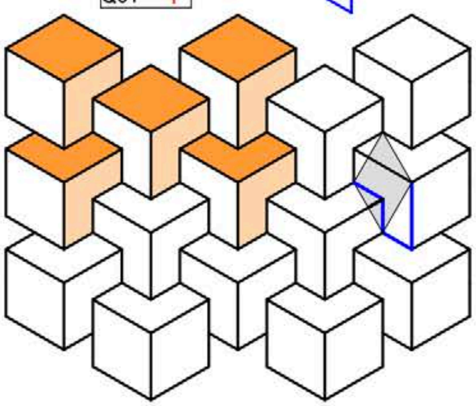


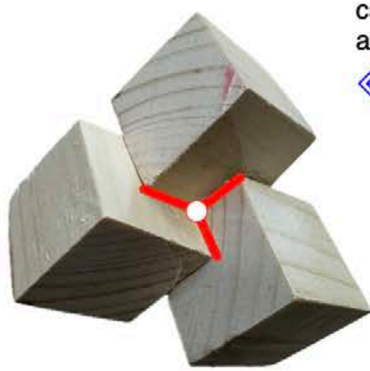




The drawings on left displays the symmetries of the projected assembly.




The axis of the 3-fold rotation goes through the intersection of the three lines at an identical angle to the three adjacent planes.





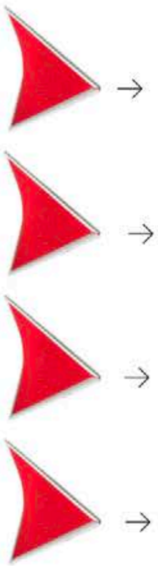
This model on the right suggests a ceremonial stair to ascend to a monument.

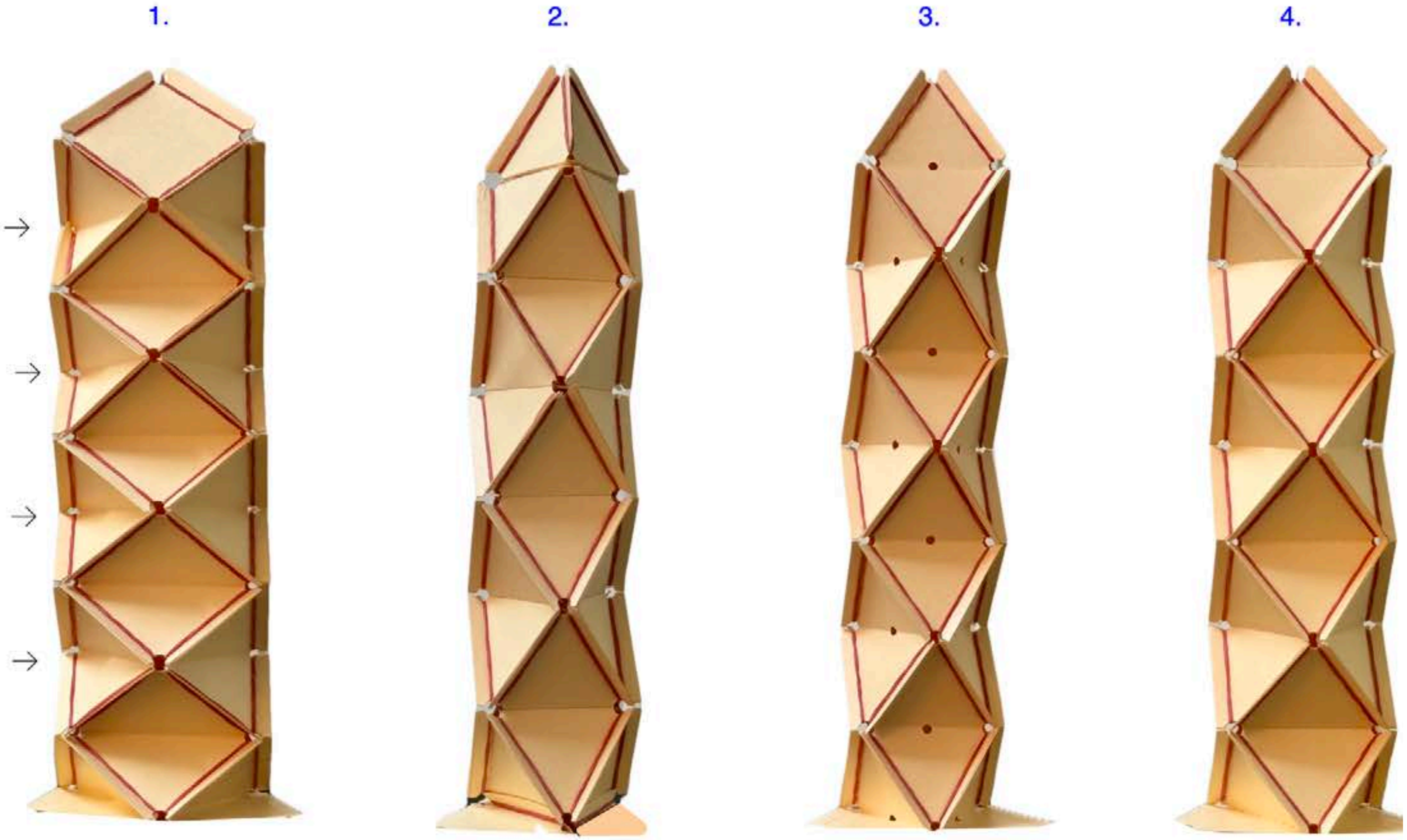
# • HP columns •

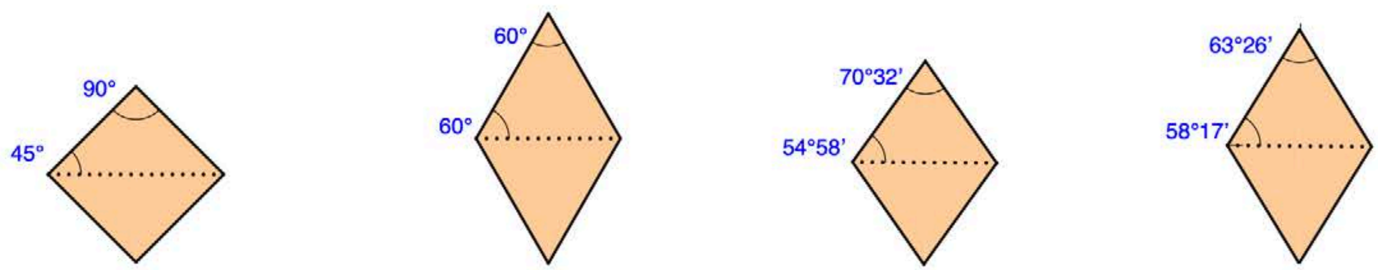
These columns are built by juxtaposing octahedra with four different triangular faces. Each face of column 1. is a half square, of column 2. is an equilateral triangle, of column 3. is the half-face of the rhombidodecahedron, of column 4. is the half-face of the rhombitriacontahedron. These faces were chosen for simplicity reasons: they are part of Poly-Kit.

These columns simulate another idea. Every pair of triangles bent along the dotted lines form a skewed quadrilateral (non coplanar). Every skewed quadrilateral determine a hyperbolic paraboloid (see P 5).

It would have been very lengthy to produce so many HP surfaces and to join them into a column for these models. So we only show some red HP surfaces to illustrate the idea.





## • Pattern I •