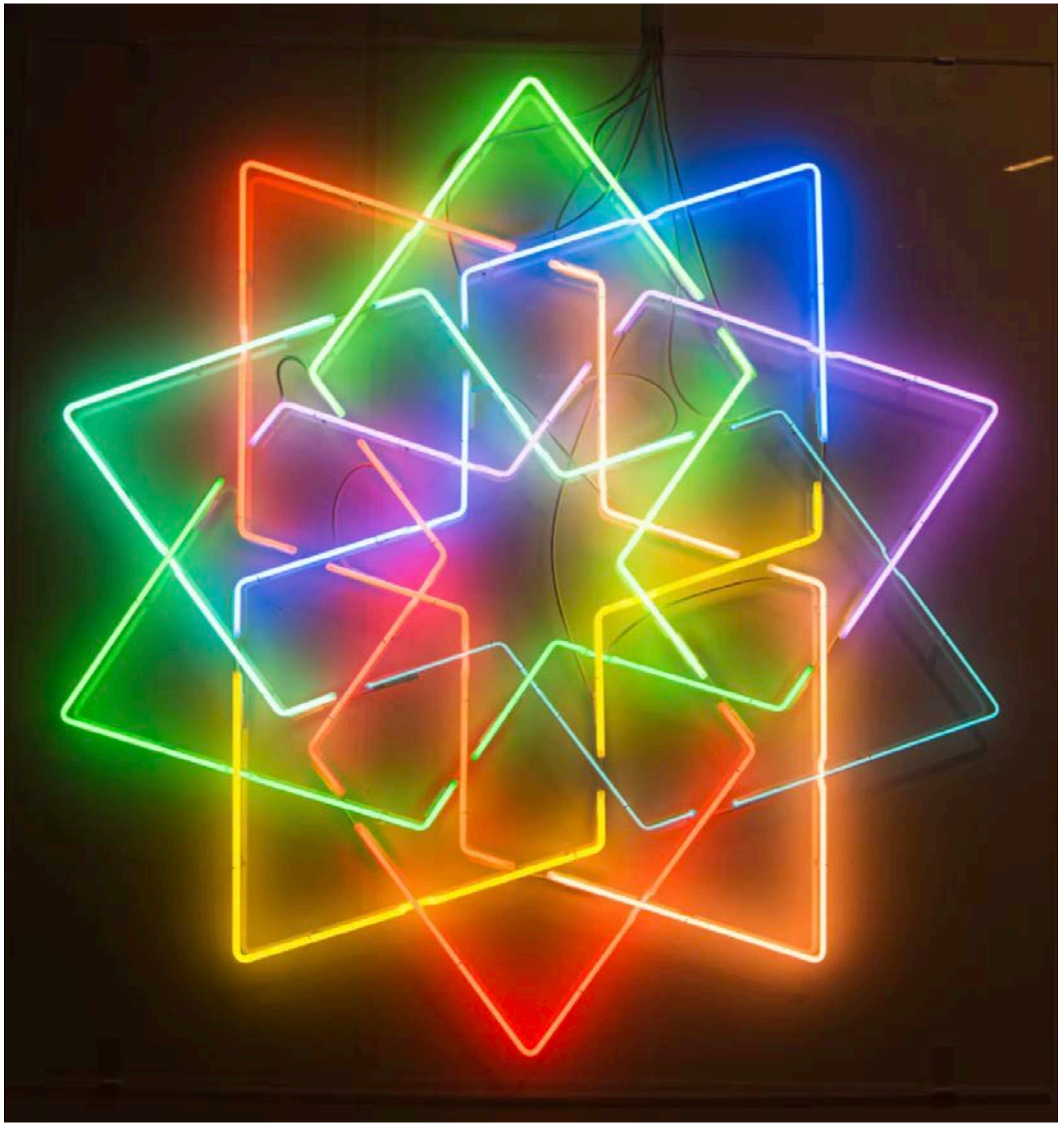
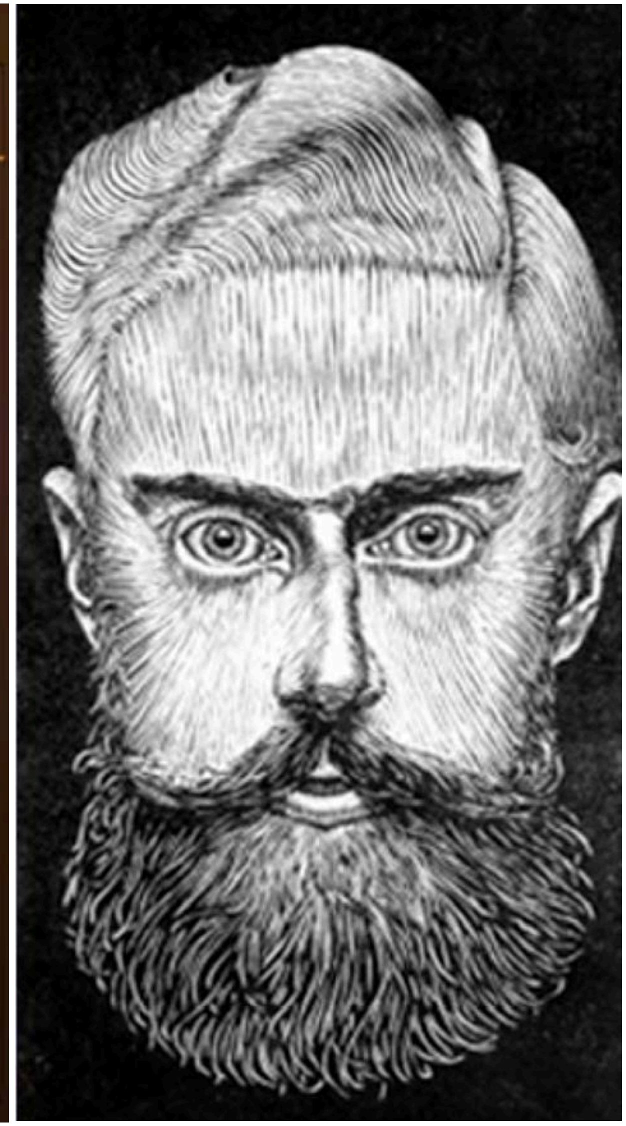
geometria2

# Regular Division of the Plane

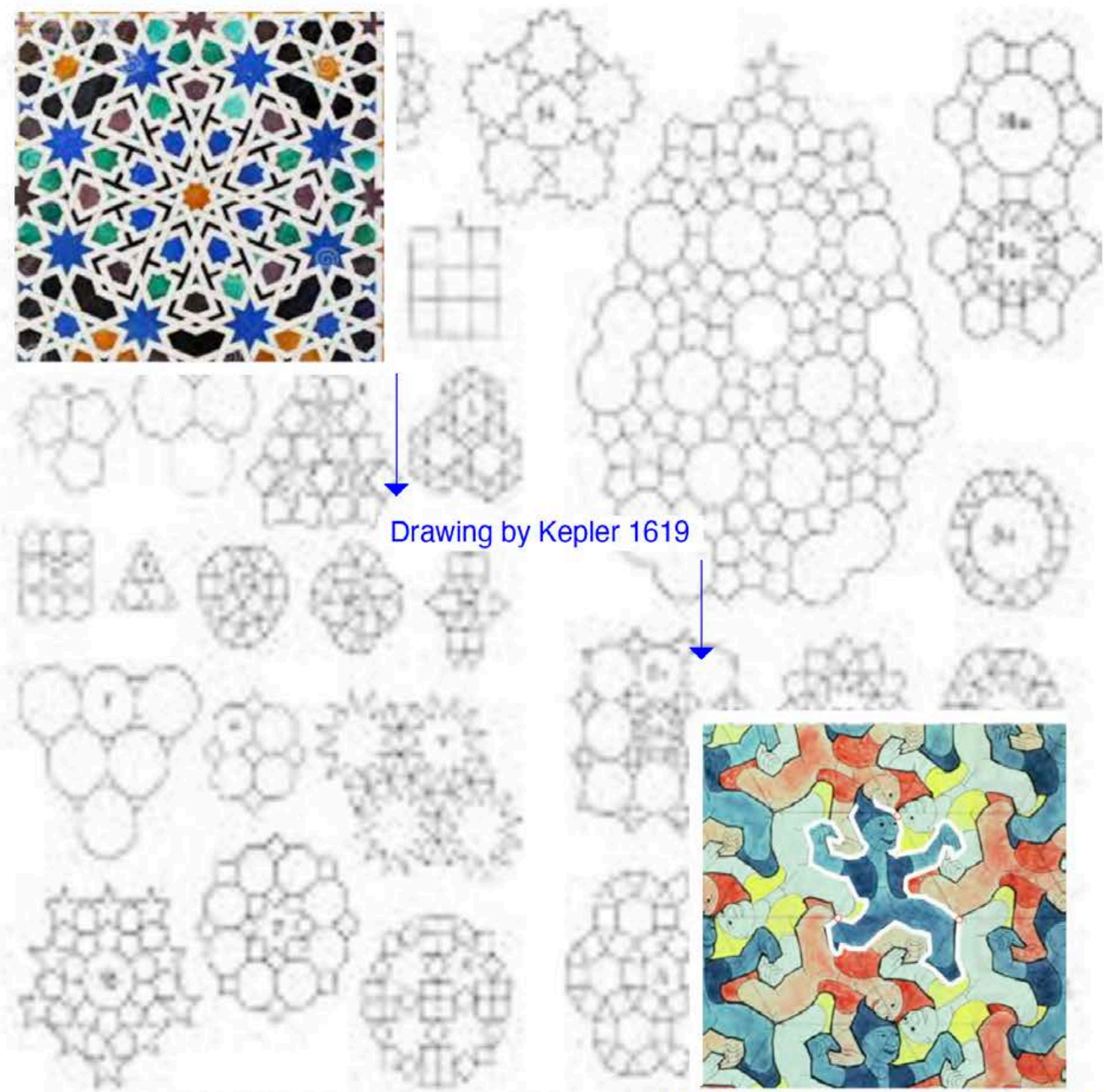




• From the Alhambra (~1333) to M. C. Escer (1898-1972) •

• Janos Baracs, Montréal 2025 •

# Regular Division of the Plane



Alhambra, Grenada ~1333

Drawing by Kepler 1619

Escher: Symmetry works 21 1938

## Regular Division of the Plane

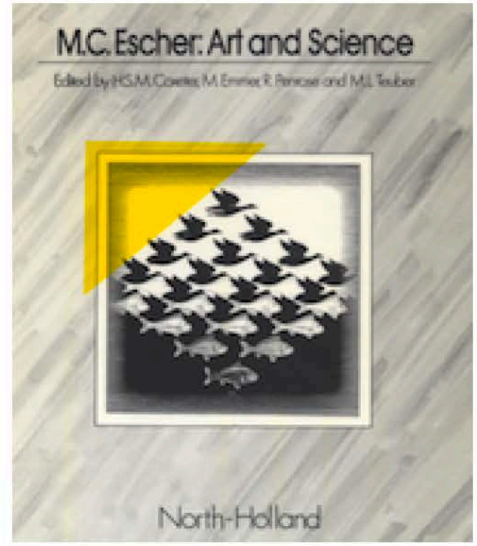
from
Alhambra to Escher

- About 50 years ago a good friend of mine helped me to discover the Dutch artist / graphic artist Escher. I am very grateful to her: Escher had a profound impact on my professional and academic life.
- Alhambra, the mosque in Grenada, Spain recognized today as a true mathematical miracle. Its Moorish ornaments contain seventeen distinct symmetry groups, which were not known to be the complete list until the end of XIX century.
- Escher first visited Alhambra in 1922 and it became an inspiration for the rest of his life. Symmetry, an abstract chapter of mathematics, is strongly linked to visual art.
- Today, we are lucky to be part of the digital revolution. We are near the completion of our drafting software Geometria 2, an excellent tool to analyze and create tilings and patterns. We hope, the following pages will help you in this endeavour.

# Regular Division of the Plane

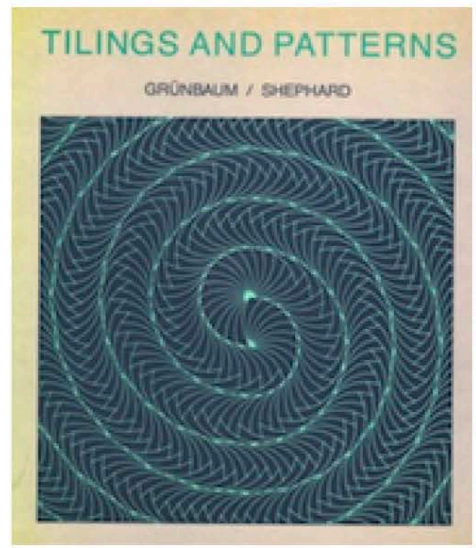
"Nous ne connaissons pas vraiment l'espace. Nous ne le voyons pas, nous ne l'entendons pas, nous ne le sentons pas. Même si nous nous tenons au milieu de l'espace et que nous en faisons partie, nous ne savons rien de lui.

**M. C. Escher**



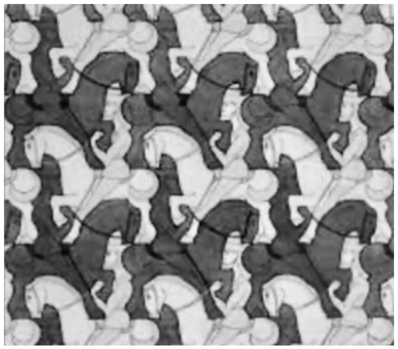

On se sent assurément rempli d'humilité quand on réalise qu'après deux millénaires d'efforts continus de la part des mathématiciens, il reste tant de problèmes simples à résoudre et tant d'inconnues à propos de cet espace tridimentionnel familier dans lequel nous vivons.

**B. Grünbaum**



# Regular Division of the Plane ; the five themes of Escher

## 1. Symmetry

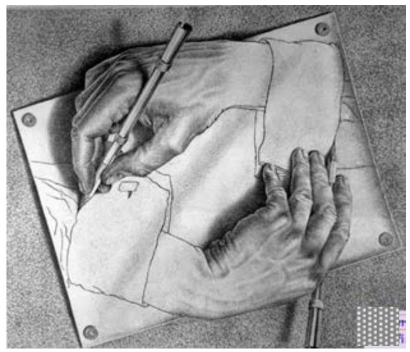


Symmetry work 67

## 2. Infinity

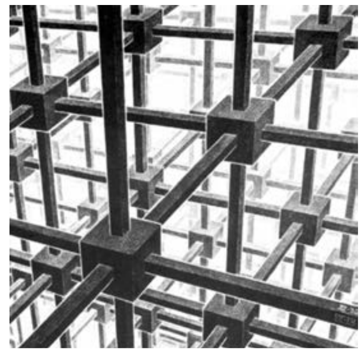
Circle Limit IV 1960

## 3. Dimensions



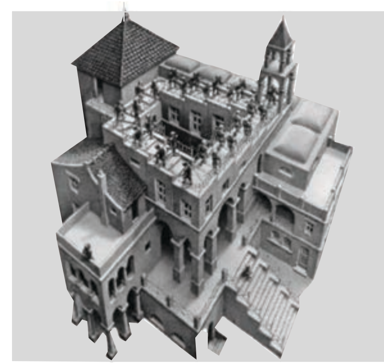
Drawing Hands 1948

## 4. Spatiality



Cubic Space Div. 1952

## 5. Perception



Ascending and Descend.

• The term “Regular Division of the Plane” was coined by Escher to describe his drawings based on covering the plane (tiling) with interlocking living creatures. He mastered th etechnic of modifying the edges of special polygons to create tessellations of recognizable figures using symmetry operations.
• In “Symmetry Work 67 the edges of a symmetrical trapezium are modified into a horseman and then subjected to two parallel glide reflections.
• It should be noted that the mathematical rules of this method were first published after Escher’s death: Tilings and Patterns by Grünbaum and Shephard in 1987.

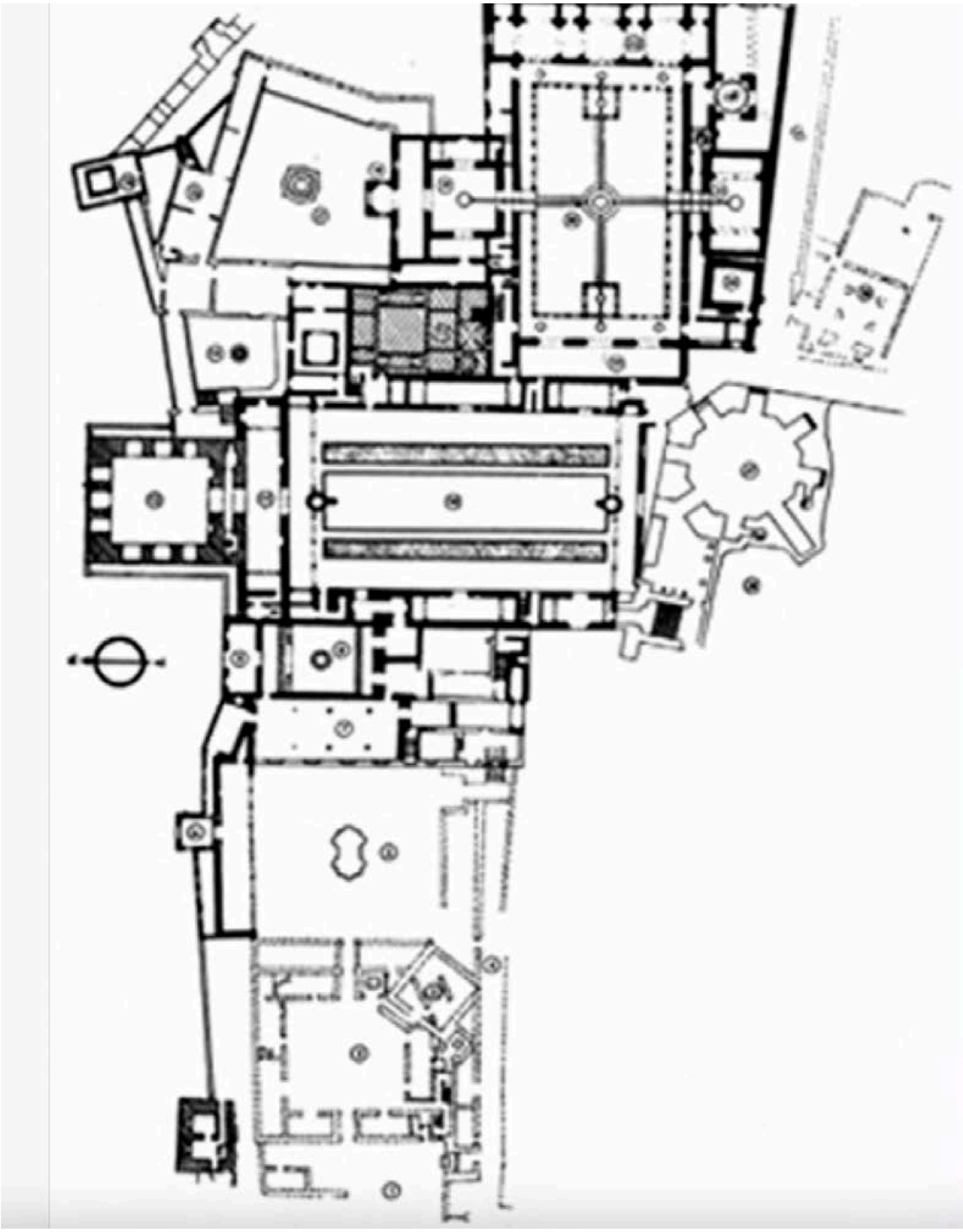
• While Escher did not have any advanced mathematical training, he demonstrated a very strong intuition in his artwork.
• Escher met the mathematician H. S. M. Coxeter in 1954 in Amsterdam and learned about hyperbolic geometry. To put it simply: it is the projection of the sphere to a plane.This new knowledge and his fascination with the concept of infinity led to produce a number of drawings, known as the Circle Limit series.
• Remember, these drawings were produced by a very clever hand before computer graphics were available.

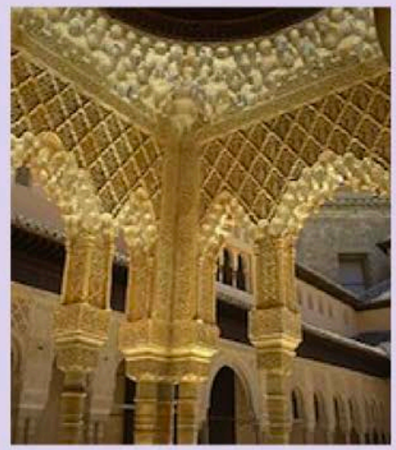
• The lithograph “Drawing Hands” (1948) is probably the most popular drawing from Escher. It depicts a sheet of paper, out of which two hands rise in the act of drawing one another into being. In the literature, It is often called a paradoxical act of drawing.
• I find this paradox truly magical. No other artist ever attempted to bridge the 2-, and the 3-dimensional universe graphically. It required a special drafting talent, geometric intuition and a robust imagination.
• This transfer is the subject of other Escher’s drawings; see the “Reptiles”, “Encounter”, and “Day and Night”

• The realistic reproduction of 3-D into 2-D is called perspective or central projection was an open problem in ancient cultures. First the artist Filippo Bruneleschi (1377-1446, and later the mathematician Girard Desargues (1591-1661) established the geometric theory of a perspective drawing.
• According to the rules, all the vertical rods meet in a very distant point below the drawing, while the horizontal rods converge in two points: one at the far left, the other one at the far right..
• This lithograph can be viewed as a synthesis of the three previous themes.
• A similar lithograph is “Depth,” made in 1955.

• The Necker cube (1832) and the Penrose Tribar (1958) are known as an impossible object. They are simple drawings of 3-D objects that do not exist in reality, according to experts.
• Escher took an interest in the subject and constructed the litograph above (1960) and some others: the “Belvedere” (1958) and the “Waterfall” (1961). The idea of these peculiar stairs is Escher’s.
• The stairway above seems to go upward and downward without gaining or losing any height. Impossible?.Not at all. The secret: the tread is not horizontal, and the riser is not vertical. It is easy to make a 3-D model; it will look exactly like the picture above!

# Regular Division of the Plane — 1. Symmetry

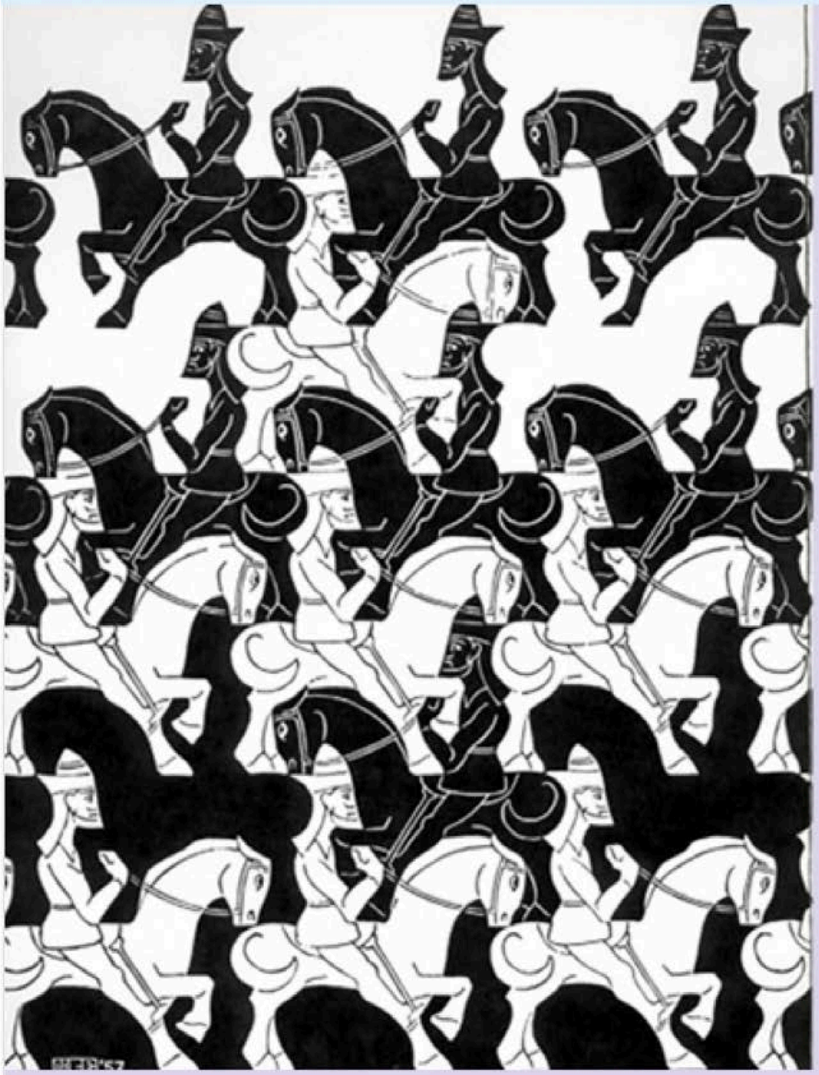




Alhambra, Espagne cca 1290

Étude sur la subdivision du plan

Étude sur la subdivision du plan

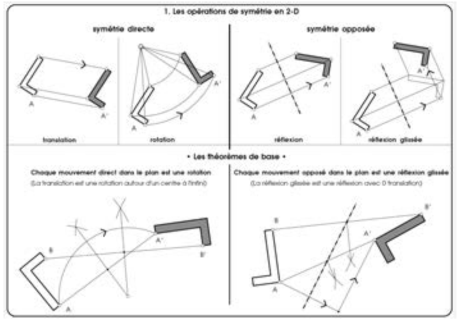


Regular Division of the Plane
Woodcut 1957

Sky and Water I
Woodcut 1938 435 x 439

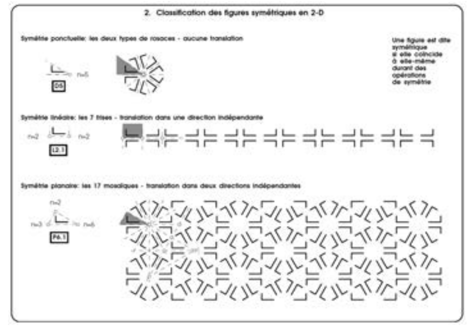
## Content of the next five pages

• A casual glance at Escher's artworks is not enough to truly understand the magic of his drawings. The following pages offer helpful geometric information to assist the viewer in appreciating his unique genius.
• The theory of symmetry is a significant chapter in modern mathematics. We recommend a book written for non-mathematicians: "Symmetry in Science and Art" by Shubnikov and Koptsik, Plenum Press, 1977.



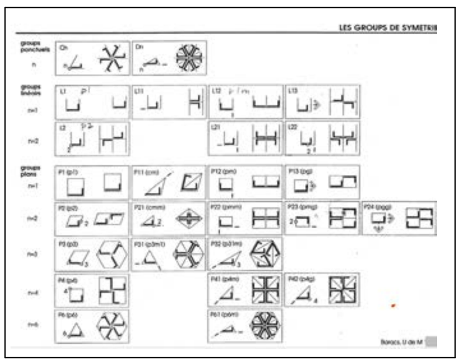
8.

The basics of the four symmetry operations.



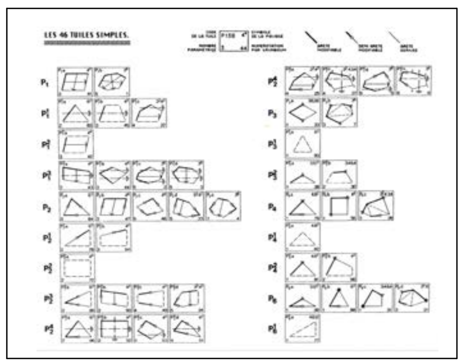
9

We present one example of the ot the three types of symmetry groups.



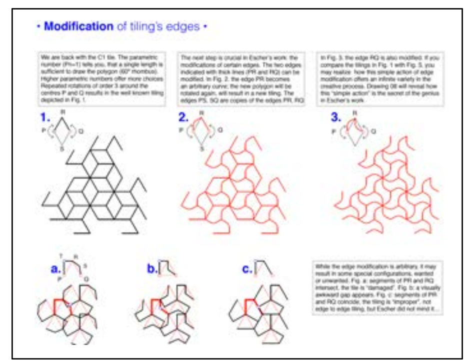
10.

Here you seea tabulated form of the seventeen plane groups.



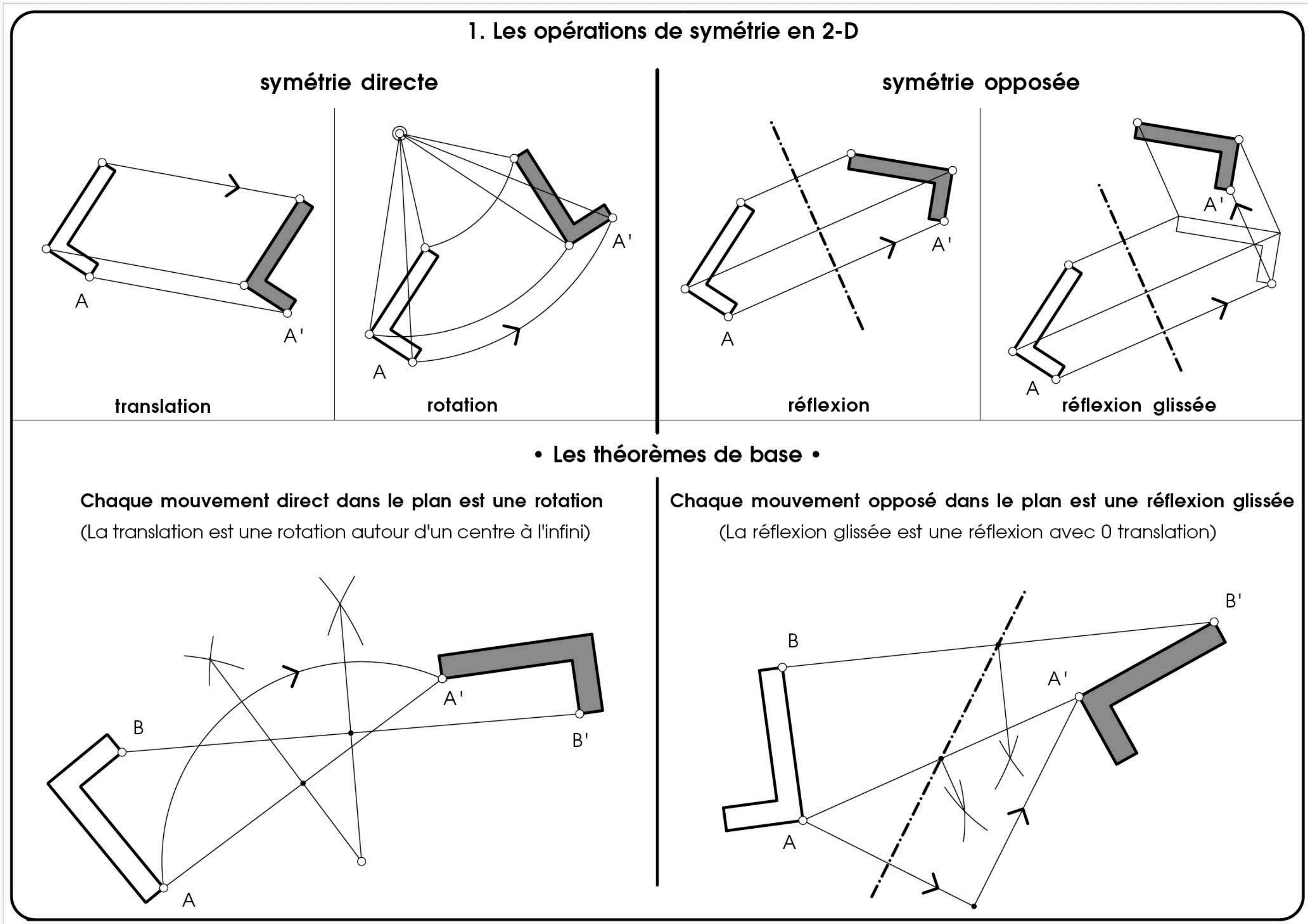
11.

This is a tabulated form of isohedral tiles: congruent copies fill the plane periodically.




12.

We picked a tile from p. 11 and showed the modification of its edges.

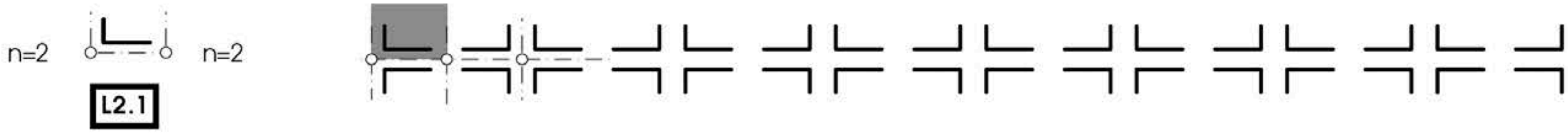

1. Les opérations de symétrie en 2-D
symétrie directe
symétrie opposée
A
A'
translation
A
A'
rotation
A
A'
réflexion
A
A'
réflexion glissée
• Les théorèmes de base •
Chaque mouvement direct dans le plan est une rotation
(La translation est une rotation autour d'un centre à l'infini)
Chaque mouvement opposé dans le plan est une réflexion glissée
(La réflexion glissée est une réflexion avec 0 translation)
B
A'
B'
A
B
B'
A'
A

## 2. Classification des figures symétriques en 2-D

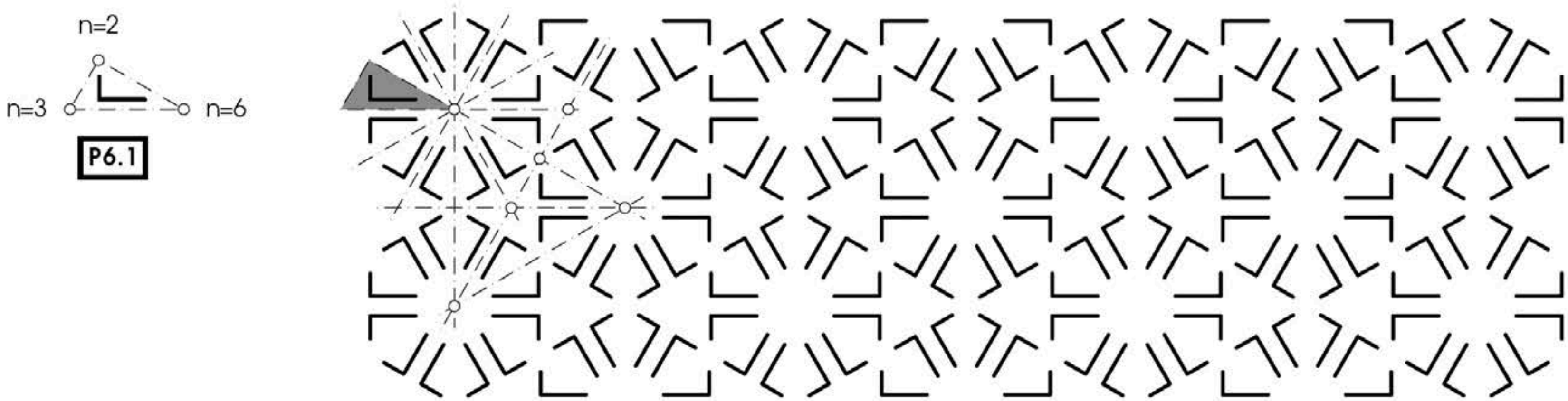
Une figure est dite symétrique si elle coïncide à elle-même durant des opérations de symétrie

**Symétrie ponctuelle: les deux types de rosaces - aucune translation**

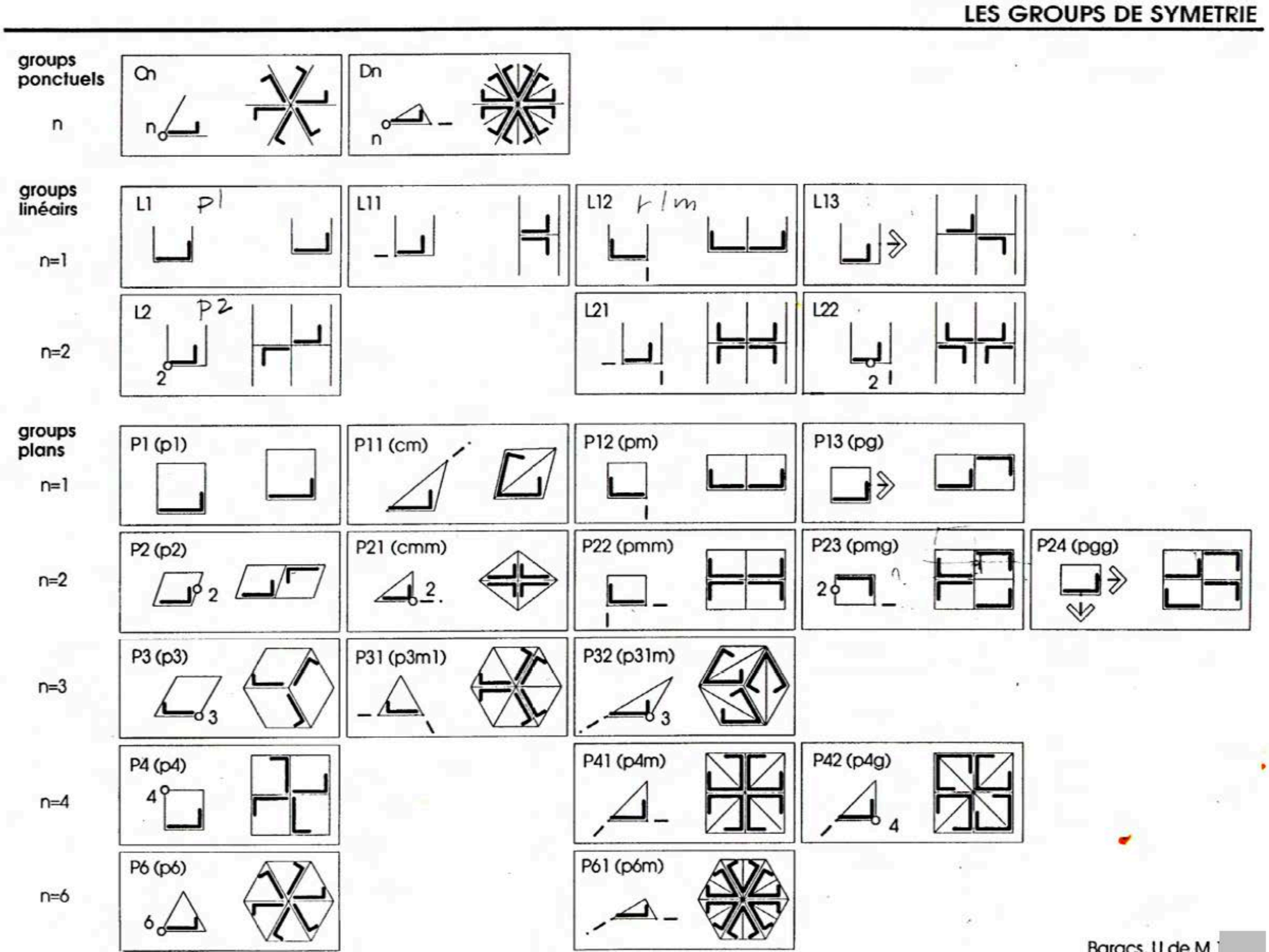

**Symétrie linéaire: les 7 frises - translation dans une direction indépendante**

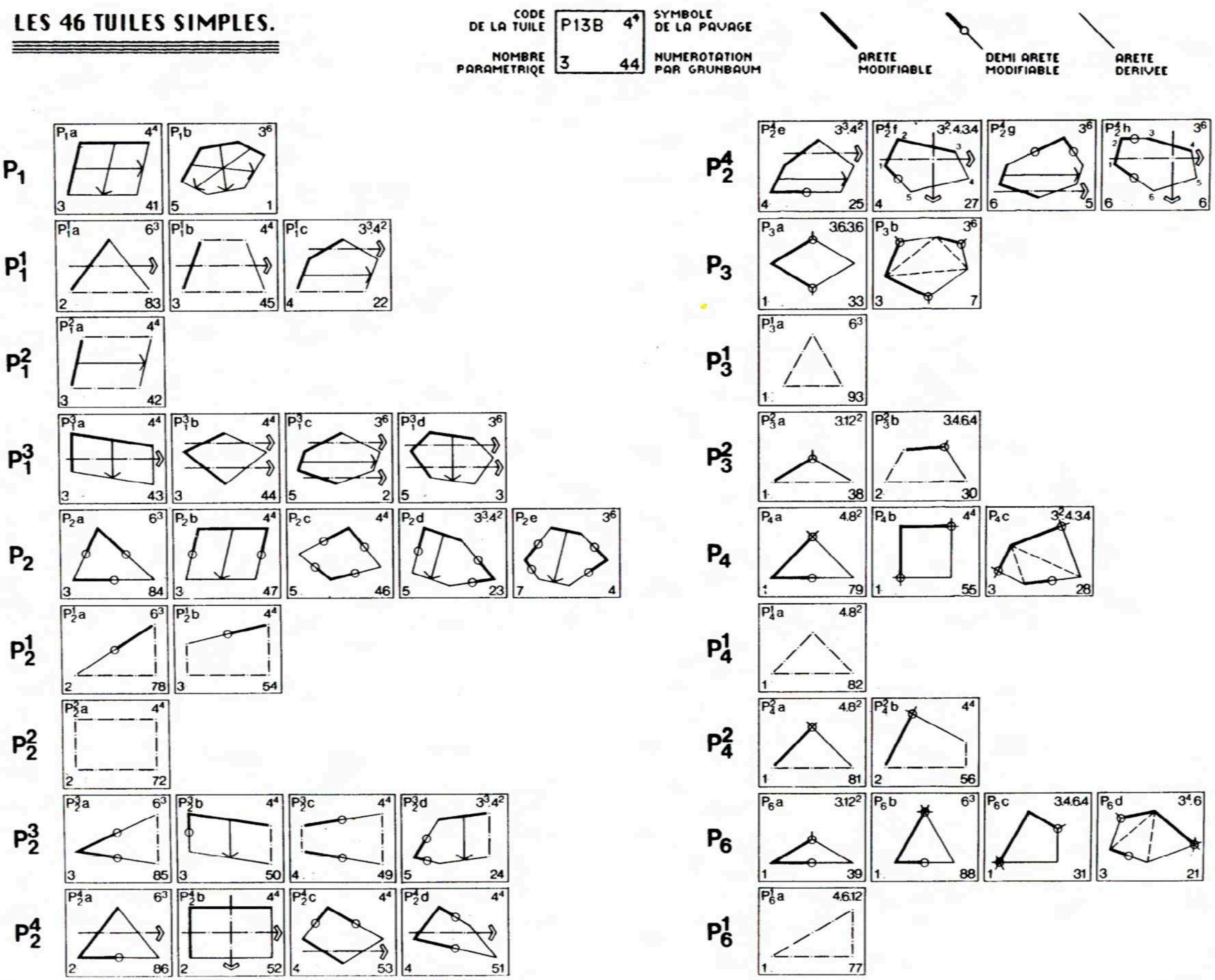



**Symétrie planaire: les 17 mosaïques - translation dans deux directions indépendantes**



# Regular Division of the Plane — 1. Symmetry





LES 46 TUILES SIMPLES.
CODE DE LA TUILE
SYMBOLE DE LA PAVAGE
NOMBRE PARAMETRIQUE
NUMEROTATION PAR GRUNBAUM
P13B
ARETE MODIFIABLE
DEMI ARETE MODIFIABLE
ARETE DERIVEE

# Regular Division of the Plane 100. 1. Symmetry

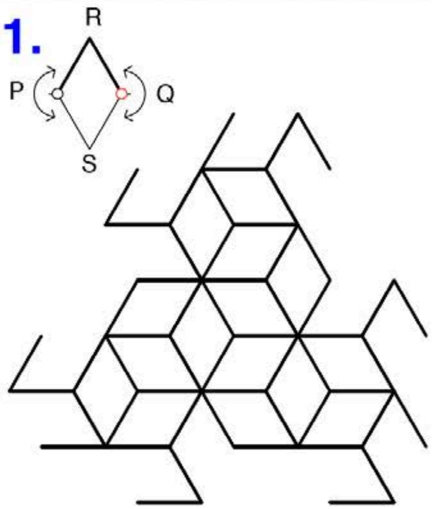
## • **Modification** of tiling's edges •

We are back with the C1 tile. The parametric number (Pn=1) tells you, that a single length is sufficient to draw the polygon (60° rhombus). Higher parametric numbers offer more choices Repeated rotations of order 3 around the centres P and Q results in the well known tiling depicted in Fig. !.

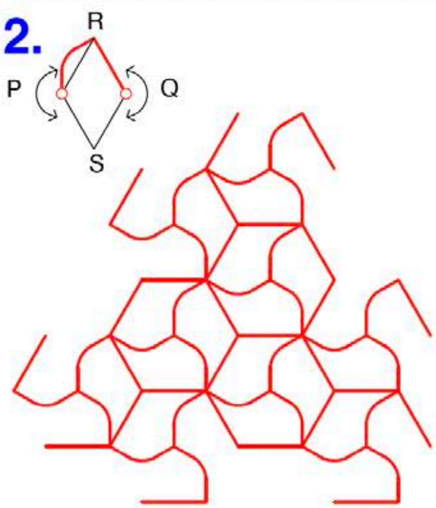
The next step is crucial in Escher's work: the modifications of certain edges. The two edges indicated with thick lines (PR and RQ) can be modified. In Fig. 2. the edge PR becomes an arbitrary curve; the new polygon will be rotated again, will result in a new tiling. The edges PS, SQ are copies of the edges PR, RQ

In Fig. 3. the edge RQ is also modified. If you compare the tilings in Fig. 1 with Fig. 3, you may realize how this simple action of edge modification offers an infinite variety in the creative process. Drawing 08 will reveal how this "simple action" is the secret of the genius in Escher's work

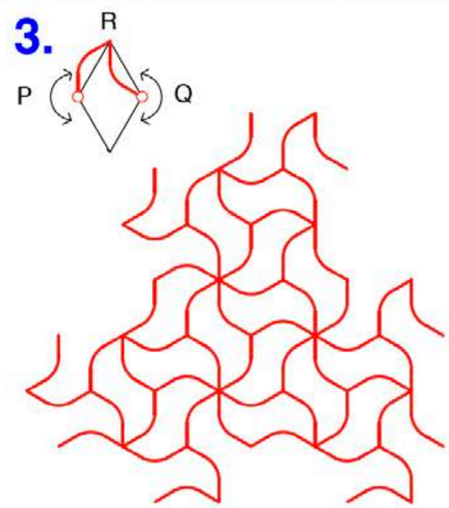
1.



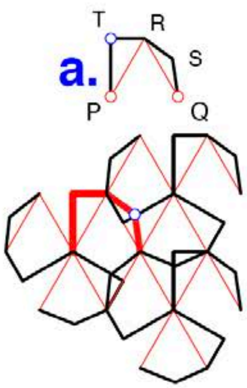

2.



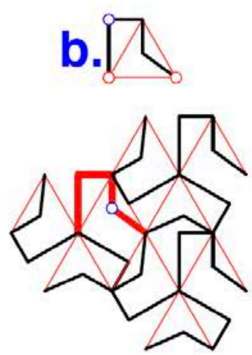

3.



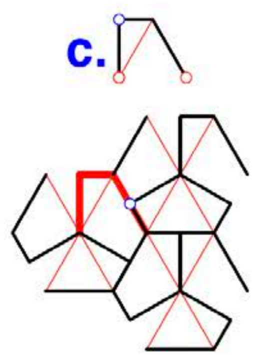

a.




b.

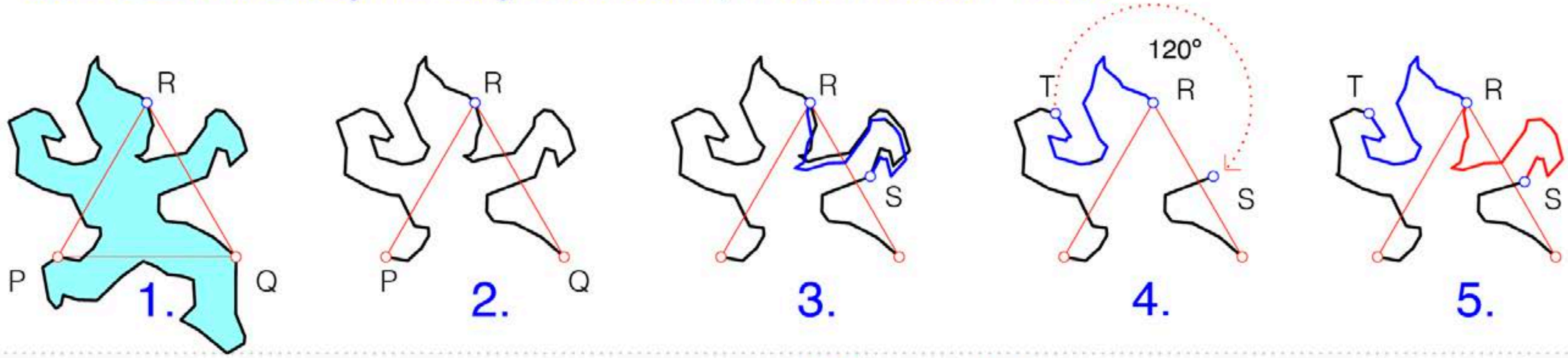


c.

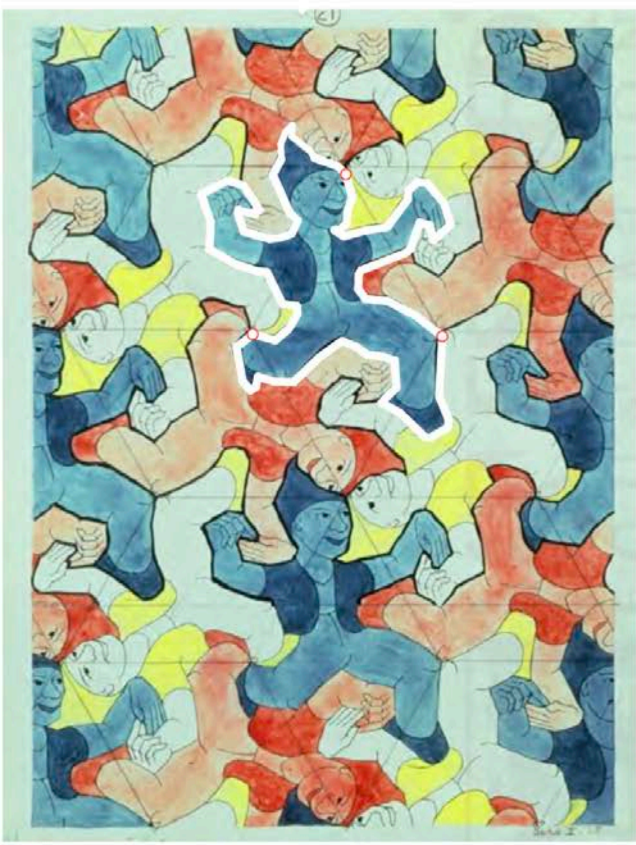


While the edge modification is arbitrary, it may result in some special configurations, wanted or unwanted. Fig. a: segments of PR and RQ intersect, the tile is "damaged". Fig. b: a visually awkward gap appears. Fig. c: segments of PR and RQ coincide, the tiling is "improper", not edge to edge tiling, but Escher did not mind it…

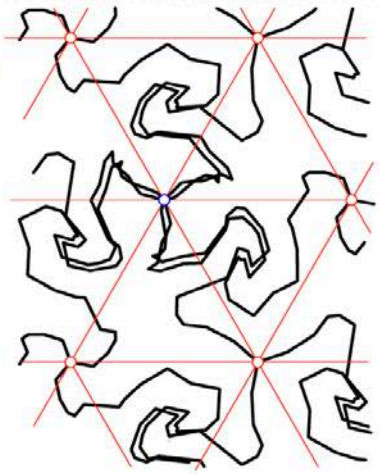
• **M. C. Escher:** Symmetry Works 21, Water colour 1938 •



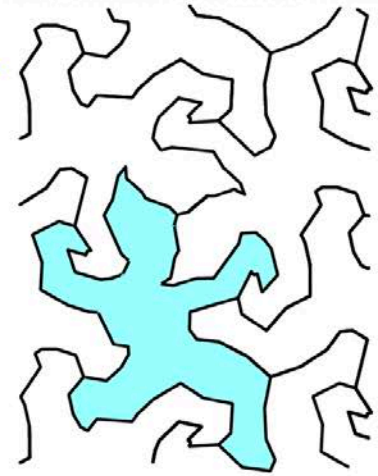

a.



b.

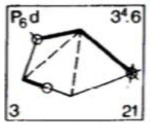


c.

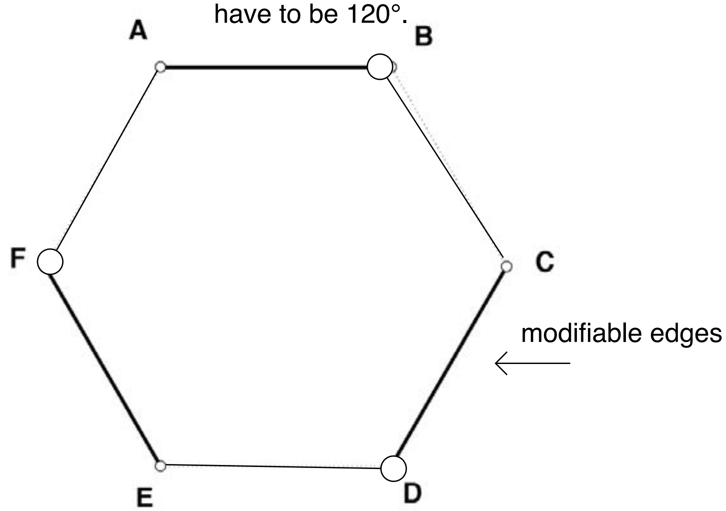


Symmetry Works 21 (ink, pencil, watercolour) was a study to prepare the more elaborate Cycle, a woodcut. I copied the motive, the "little man" (Fig. a and 1), and the rhombic grid. It shows that the tile C1 was used. PR and RQ are the independently modified edges (Fig. 2). After the rotations, the result is the tiling in Fig. b. The gap is seen in Fig. 3. To eliminate the gap, Escher rotated the path TR 120° into RS (Fig. 4), the result is Fig. 5 and Fig. c, no gaps, no intersections in the tiling. *The creation of the modified edges and to foresee the consequences in the tiling, to visualize the two "sides" of a path is truly brilliant!*

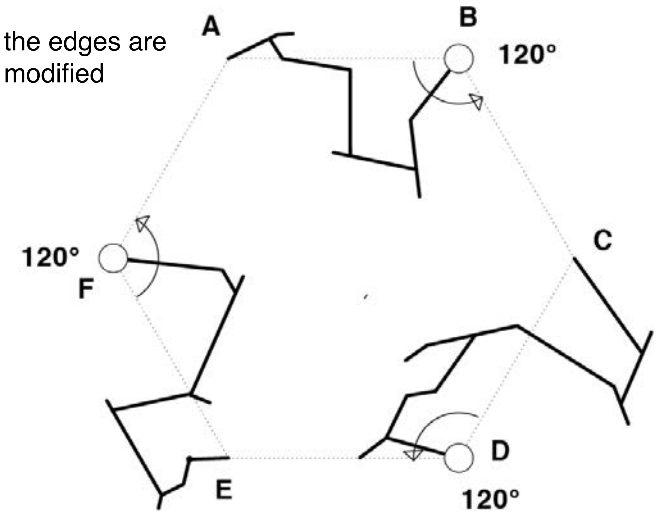
1. the selected tile



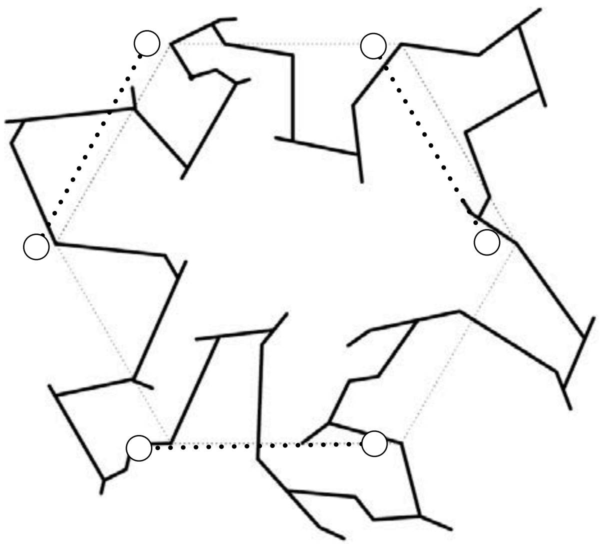
2. the angles B, D, and F have to be 120°.



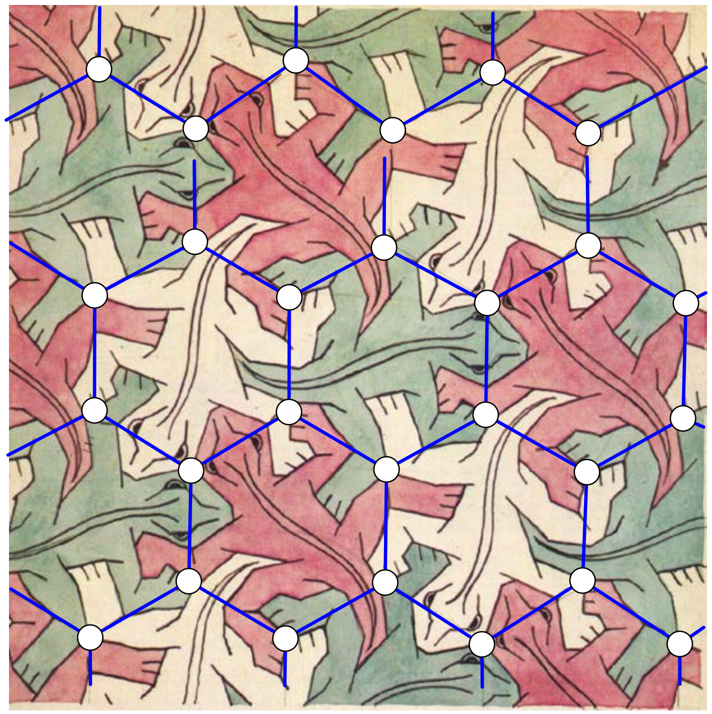

3. the edges are modified




4. the modified edges rotated 120°



5. the result



Construction of the “Reptiles”

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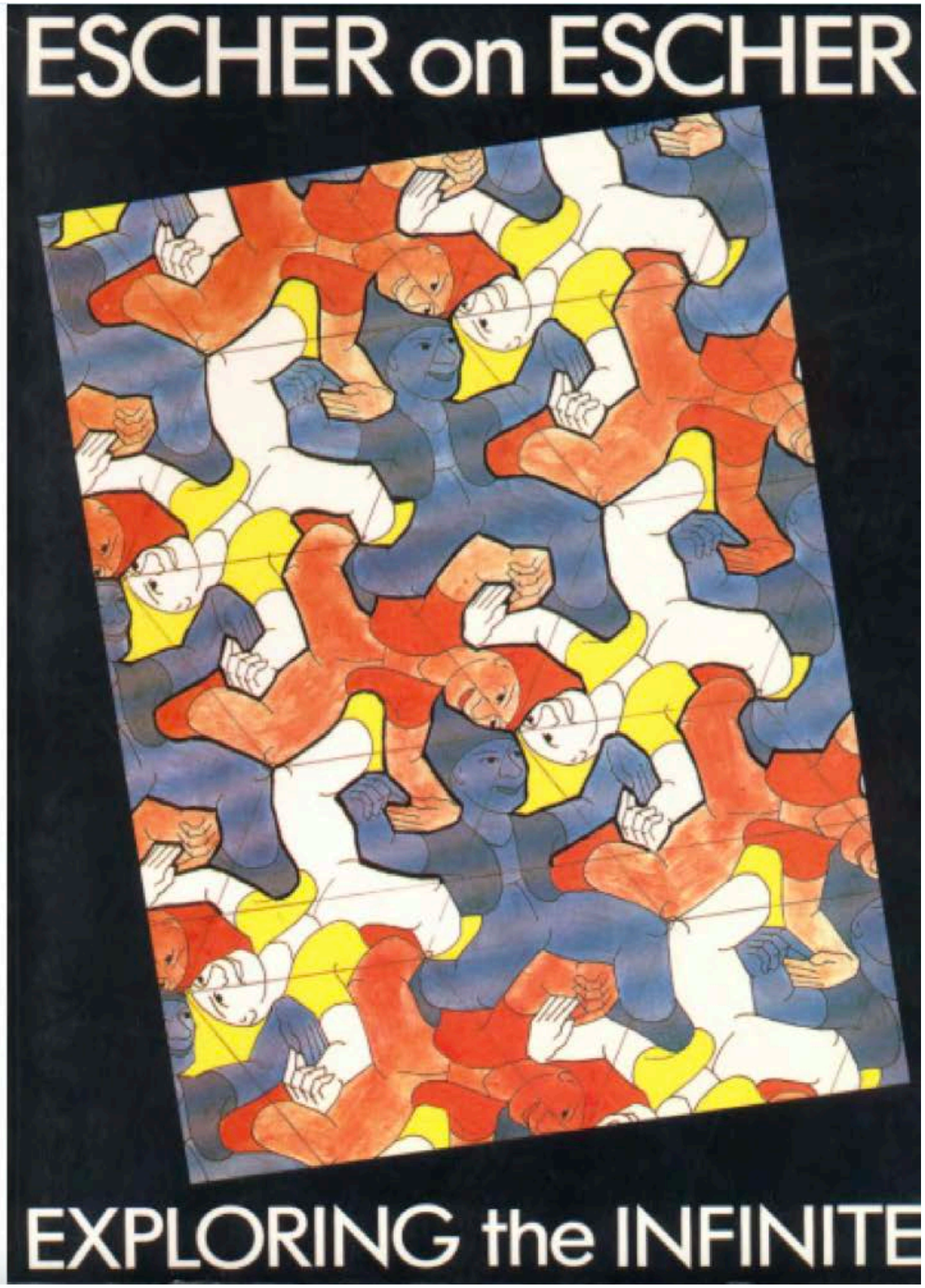
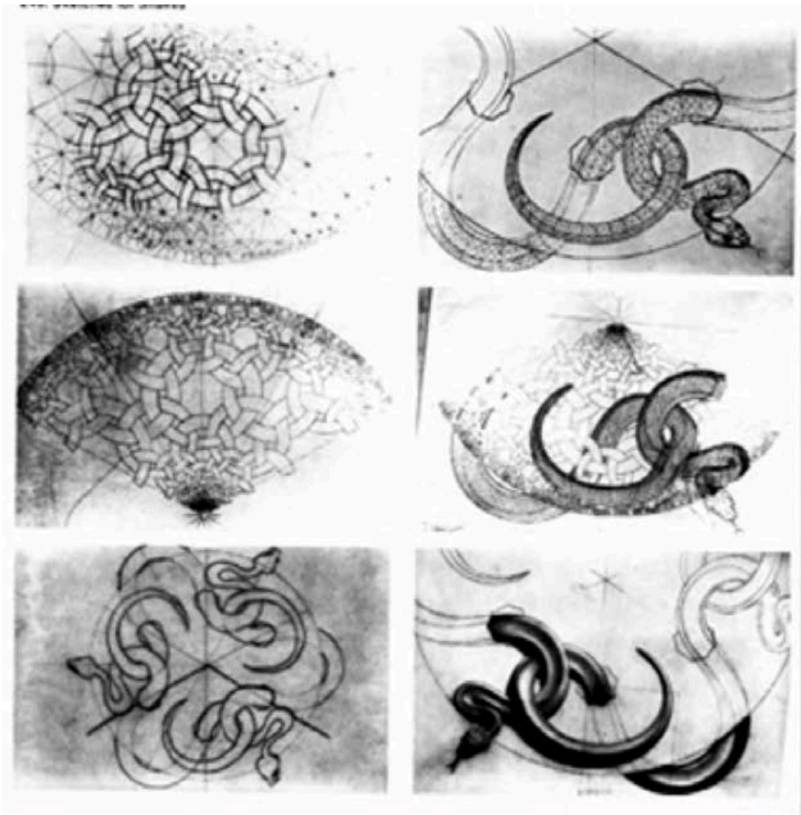
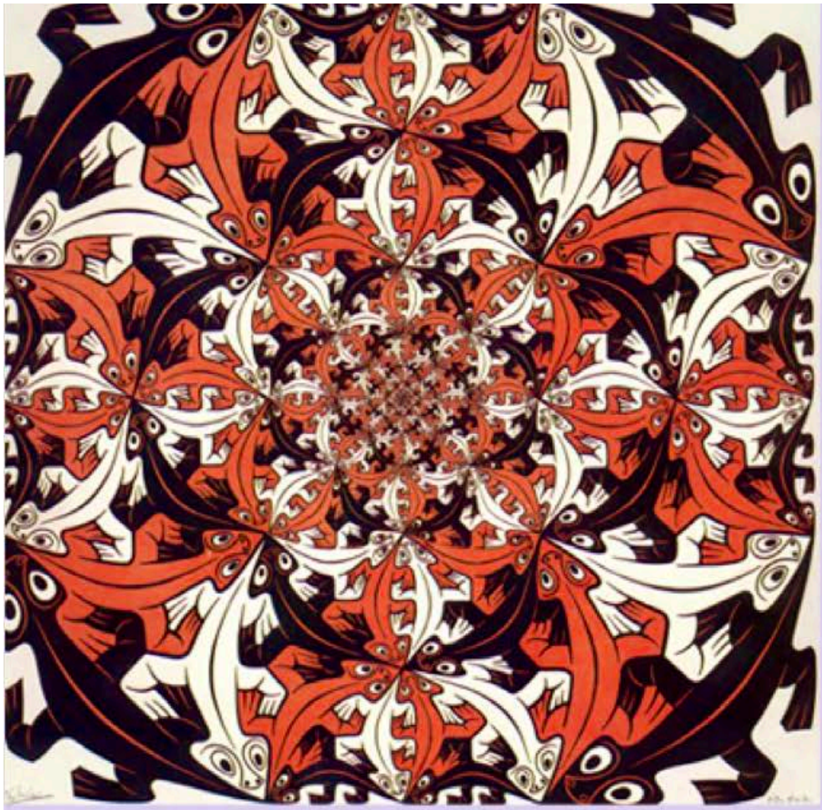




Snakes
Woodcut 1969 498 x 447



Circle Limit III
Woodcut 1959 diam. 415

Smaller and Smaller
Woodcut 1956 380 x 380

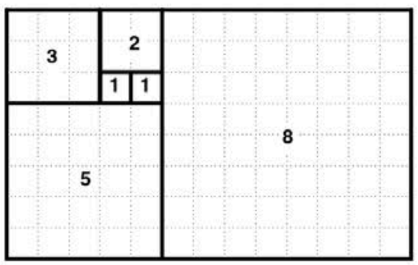
## • Les nombres Fibonacci - une série infinie •

**Valeurs de départ:** $F_0 = 0;\ F_1 = 1$ **La formule:** $F_n = F_{n-1} + F_{n-2}$

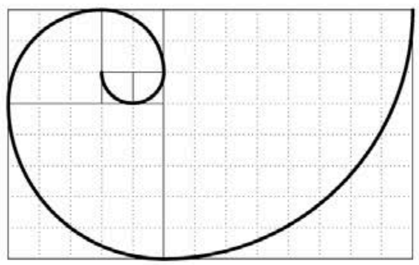
$\boxed{0,\ 1,}$ 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, ...

$F_n / F_{n-1} \longrightarrow \phi$ le nombre d'or $\dfrac{1+\sqrt{5}}{2} = 1.618...$

2/1=2.000, 3/2 =1.500, 8/5=1.600, 13/8=1.625, 21/13=1.615, 34/21=1.619 ...

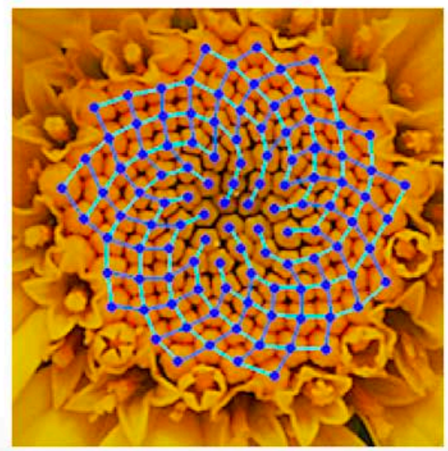


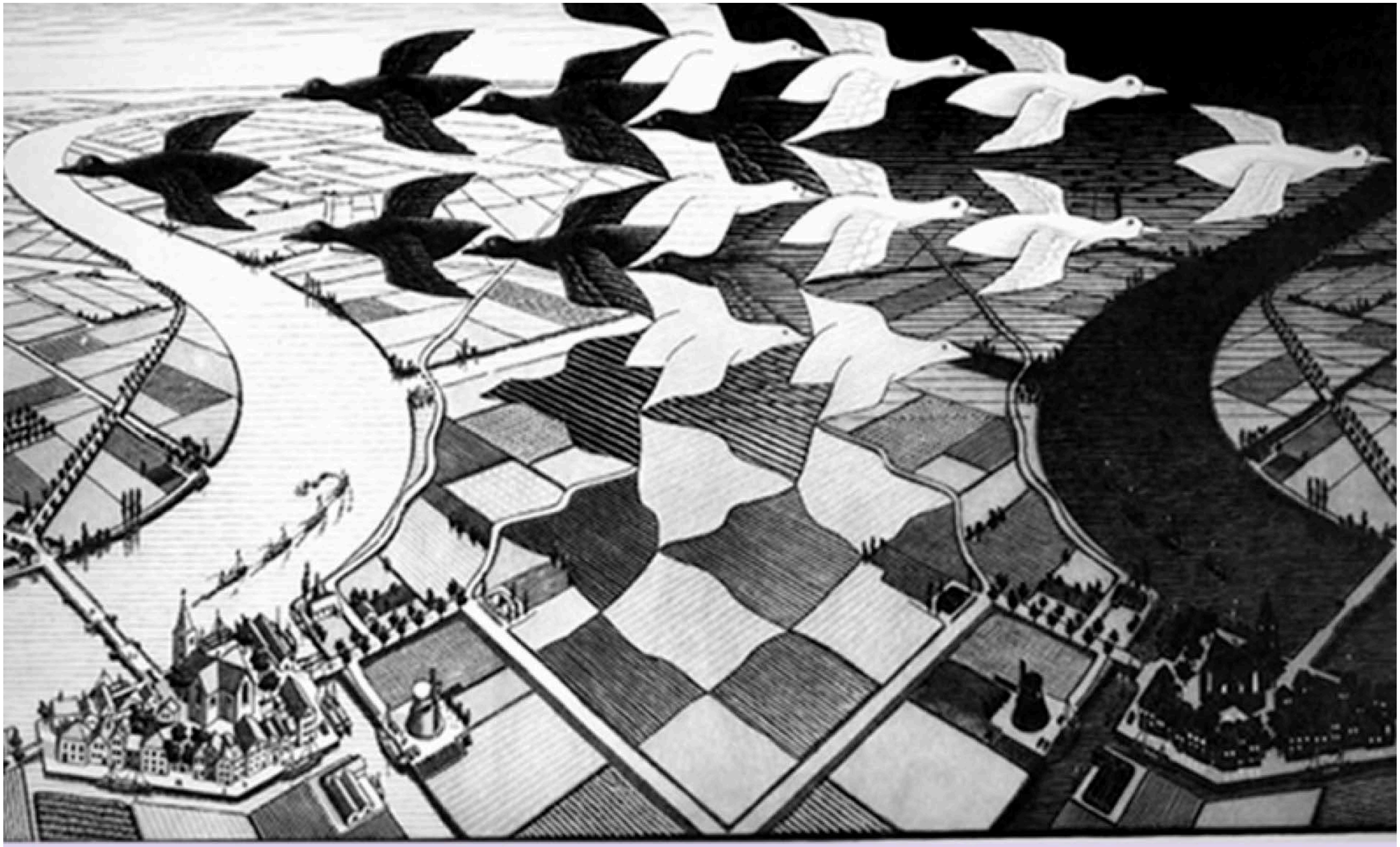

Les tuile Fibonacci



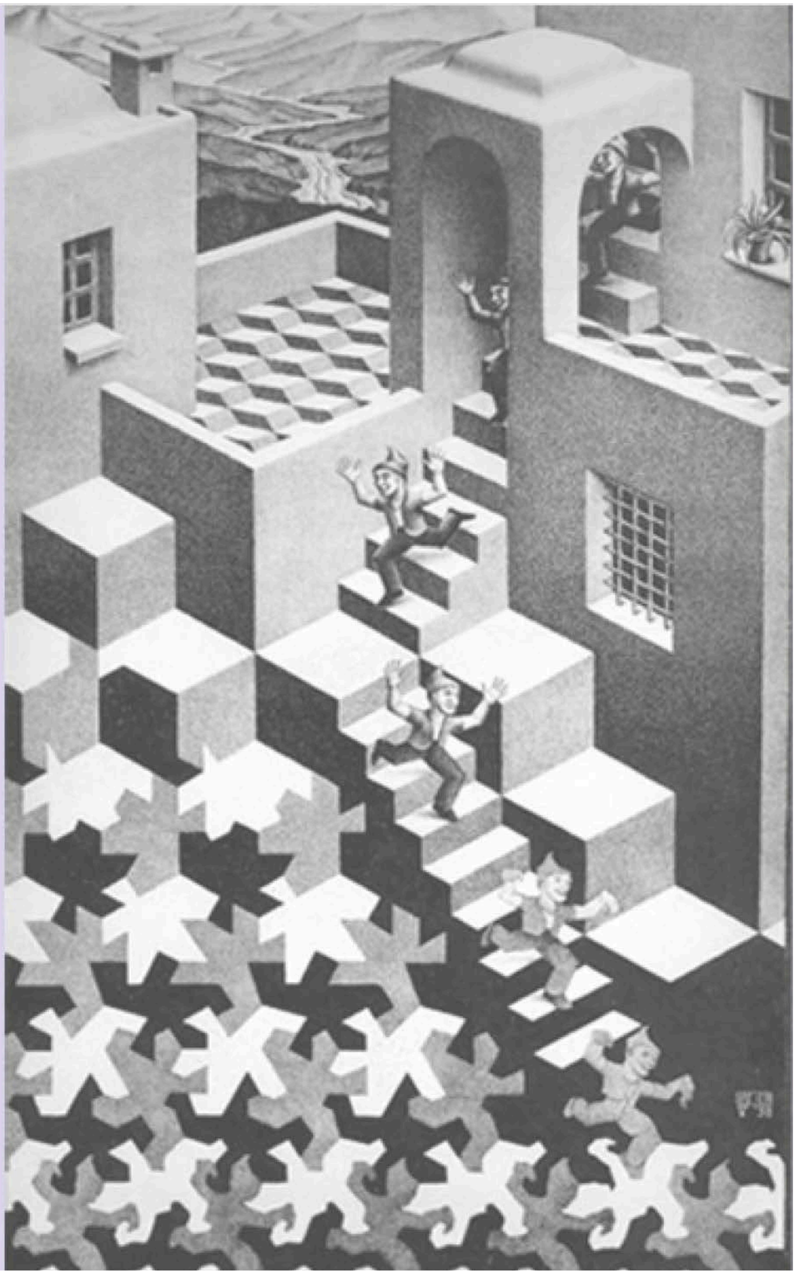
La spirale Fibonacci

Yellow Chamomile



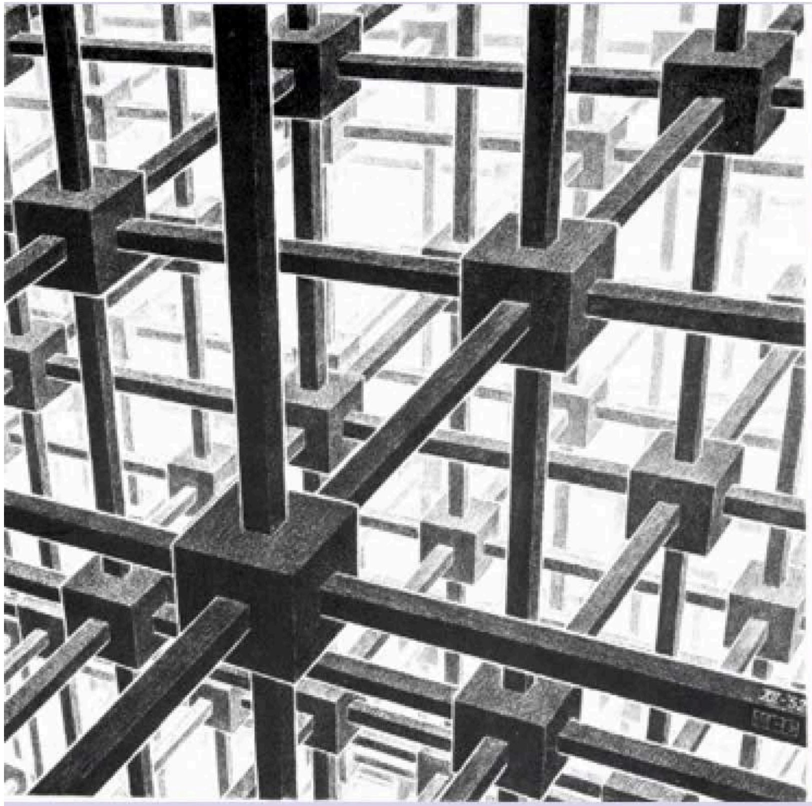


Day and Night
Woodcut 1938 391 x 677



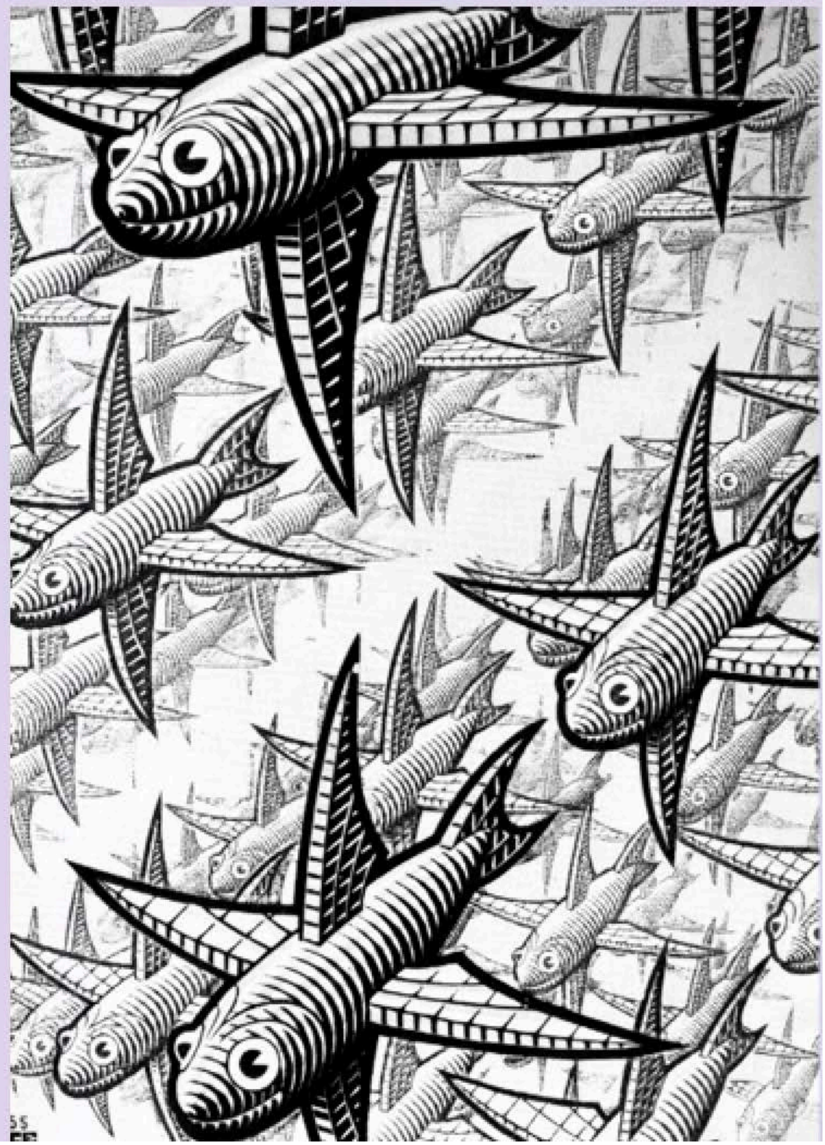
Cycle
Lithograph 1938 475 x 279

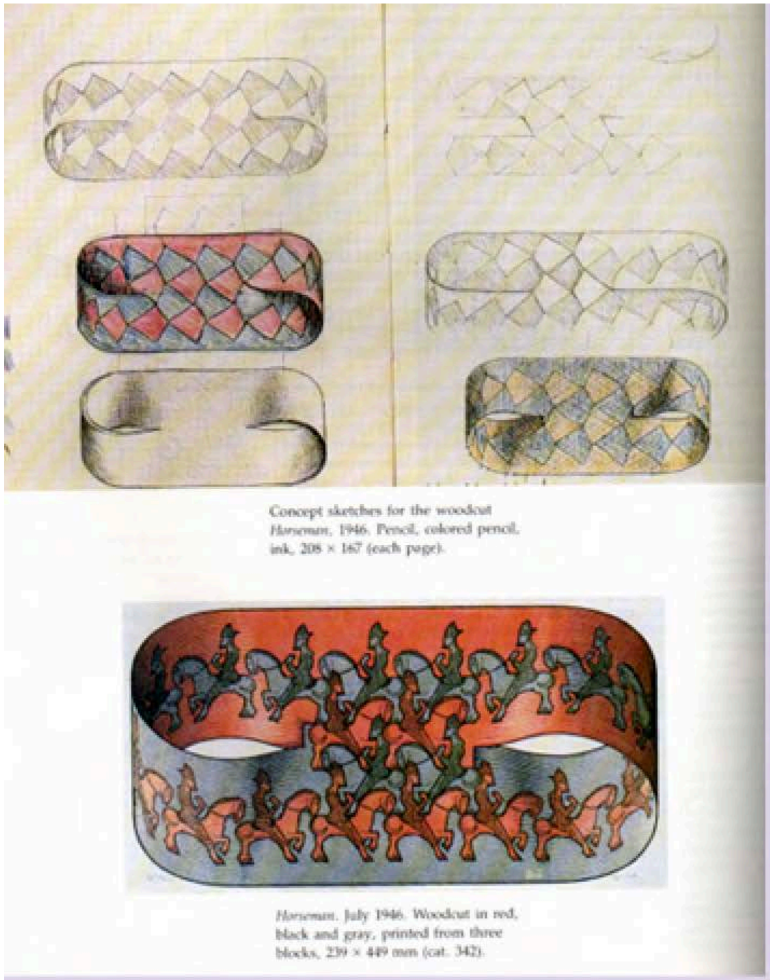
**Reptiles**
**Lithograph 1943**



Cubic space division
Lithograph 1952 266 x 266

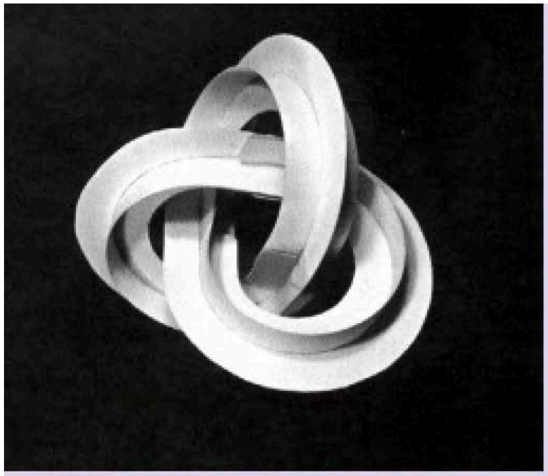
Depth
Woodcut 1968 320 x 230



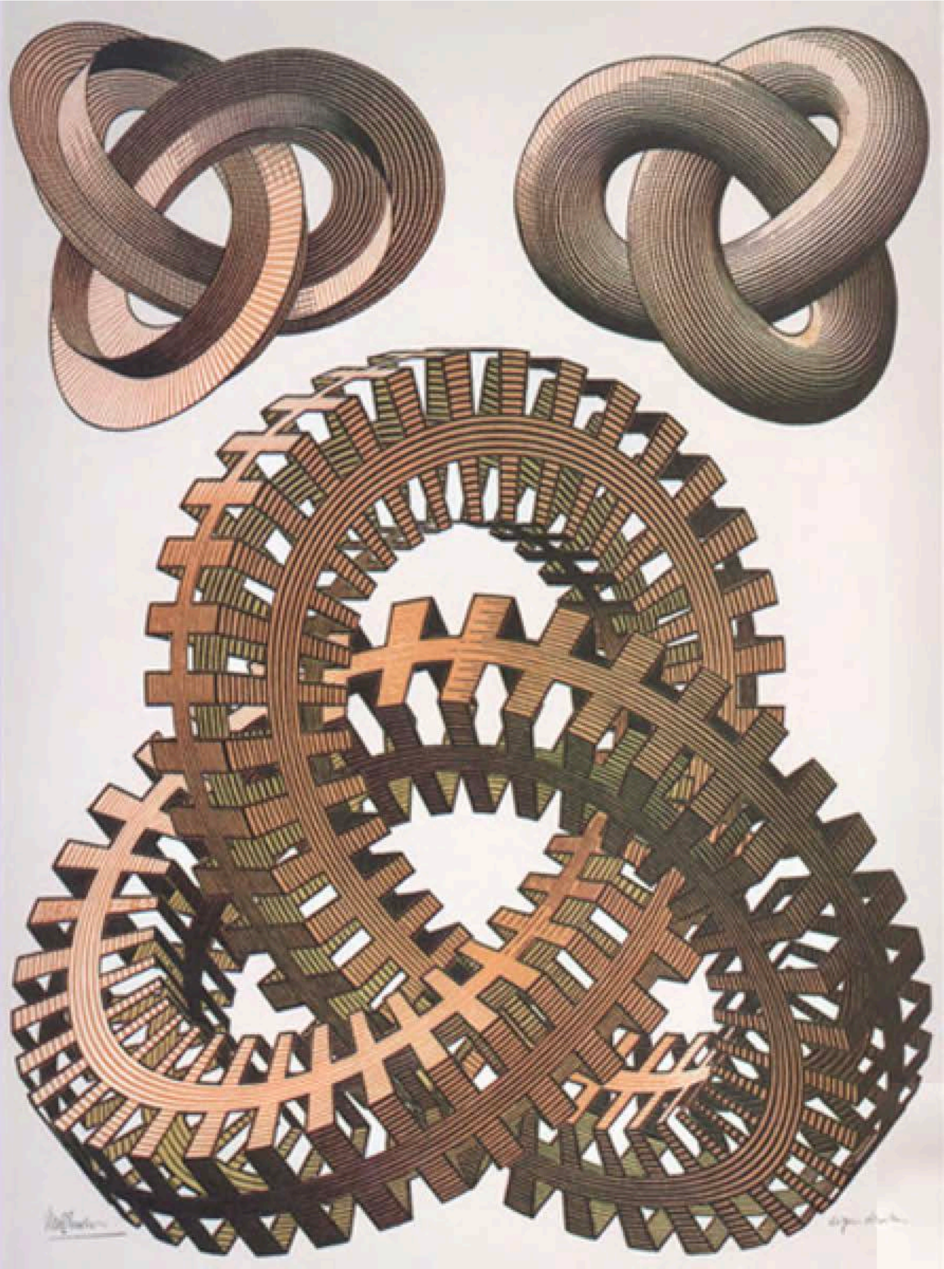



Horseman
Woodcut 1946 239 x 449

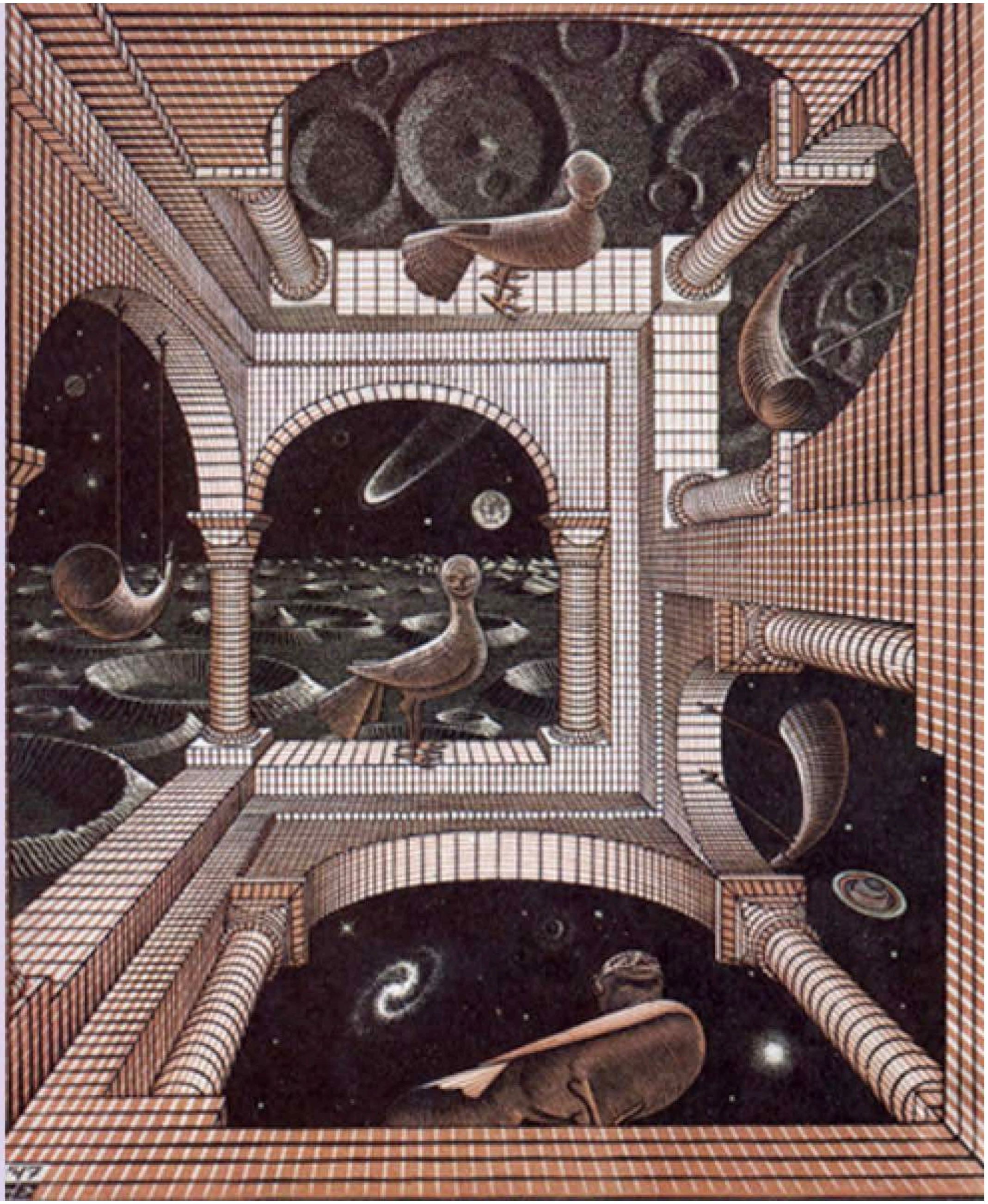
Möbius Strip
Woodcut 1963 453 x 205



Cardboard model

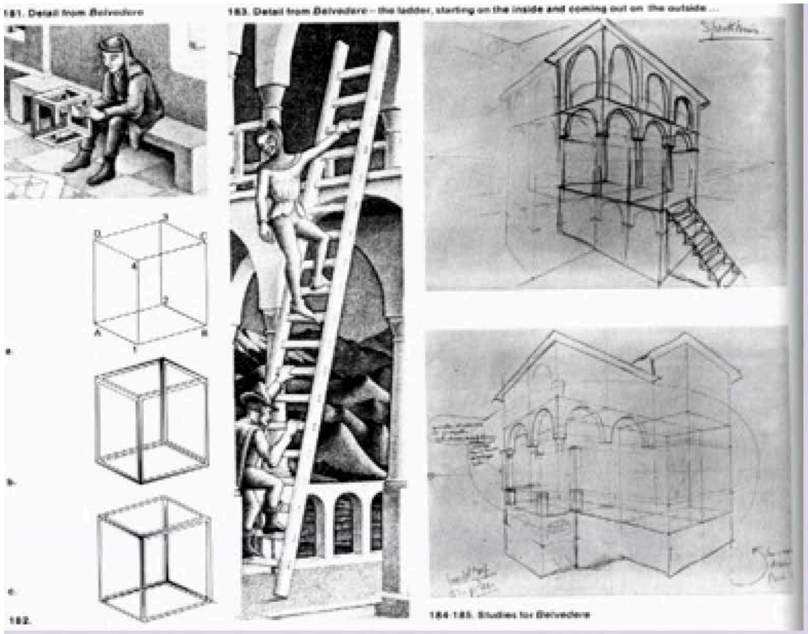


Knots
Woodcut 1965 430 x 320

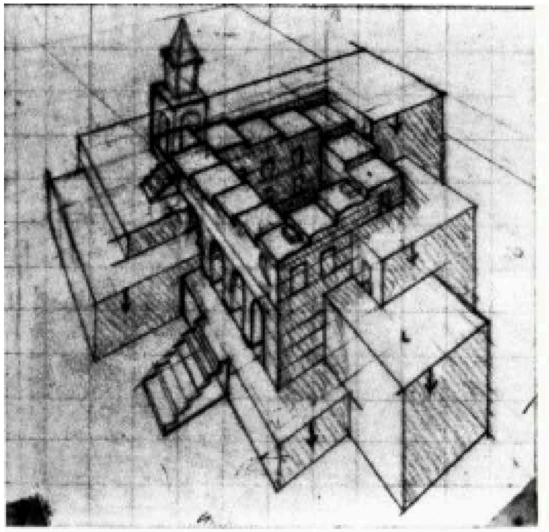


Other World
Woodcut 1947 318 x 261

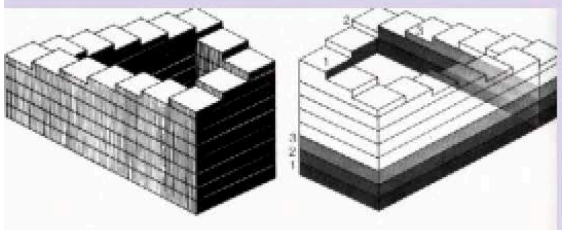
# Regular Division of the Plane 5. Perception




Belvedere
Lithograph 1958 462 x 295



201. Preparatory sketch for *Ascending and Descending*



Waterfall
Lithograph 1961 380 x300