geometria2

Poly-Kit

- I hear, I forget
- I see, I remember
- I touch, I understand

• An educational kit to enhance spatial abilities and creativity •

• Janos Baracs •

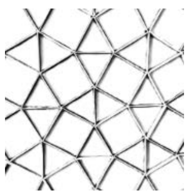
Themes

Pages

The first version of Poly-Kit I designed was published in 1979 In Montreal. It was widely used in elementary, high-school and universities and by students and professionals. This second edition is a vastly improved version with many improvements based on years of experience with different participants in various establishments.

This new Poly-Kit contains sixty identical die-cut cardboard sheets with a carefully selected set of polygons to be attached by rubber bands. They are designed to assemble three families of geometrical configurations:

• regular and semi-regular tilings (tesselations)



• polyhedral surfaces (a new spatial concept)

• regular, semi-regular and some dual polyhedra

We claim that Poly-Kit can be an enjoyable toy or a recreational product or a spatial design tool for all ages. The juxtaposition of regular polygons in the plane or space is an excellent route to discover the beautiful order in space and helps to enhance our perception of our 3-dimensional universe.

This manual is written for:

- parents to help their young child assemble simple models
- teachers (elementary, high school or college) to organize class activities (see five Exercises)
- architects, designers, engineers, artists to explore new options in their field.

We include definitions, geometrical terms and the lists of a finite number of possibilities with certain rigour while avoiding the esoteric language of mathematics.

Symmetry is a visual and attractive chapter in mathemat-; ics; also a neglected subject in schools. Pol-Kit is a hands-on tool for discovering and applying symmetry operations in 2-D and 3-D.

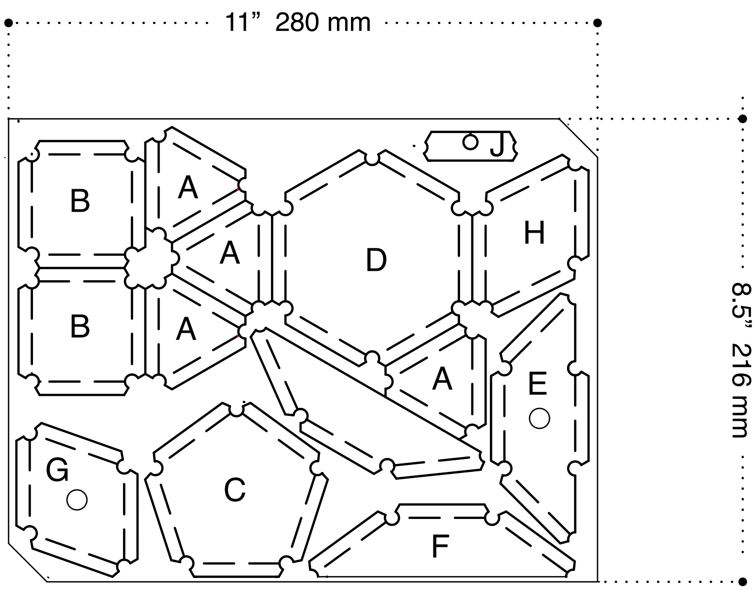
The forms you create with the kit may not land directly into practical solutions; they are the basic building blocks of space. They come to life if the users recognize the options to transform, attach, dissect, and sculpt forms to respond to specific criteria. Here, the limit is your imagination and your creativity.

We hope to receive comments from the users on how to improve the next edition.

The kit consists of sixty identical sheets of light cardboard with 14 figures die-cut on each sheet. The figures are four regular triangles, two squares, a regular pentagon and a hexagon. To save materials, a regular octagon, a regular decagon and a regular dodecagon are assembled from components as shown on the right.

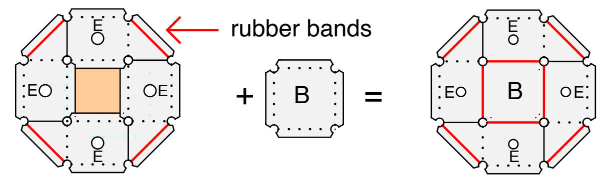
The assembly of a polyhedron is simple, fast and precise. You may pop out ten polygons at once with a little pressure, and once the scored tabs have been bent, adjacent tabs are fastened with the rubber bands.

The kit is sufficient to build the 5 regular, the 13 semi-regular and 2 dual polyhedra, some prims and antiprisms.



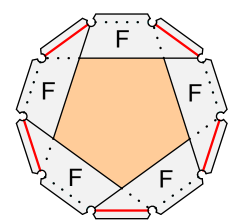
The figure J is included to help with the display by suspension of a polyhedron.

An octagon comprises four figure F’s attached with four rubber bands. The decagon is built with five figure E’s, the dodecagon is made of six triangles and six squares. The octagon maybe completed with a square, the dodecagon with a hexagon.

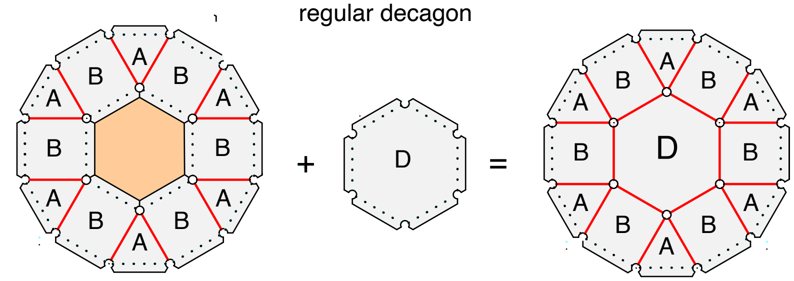


assembly of a regular octagon

octagon with a square attached



assembly of a regular decagon

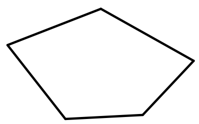


assembly of a regular dodecagon

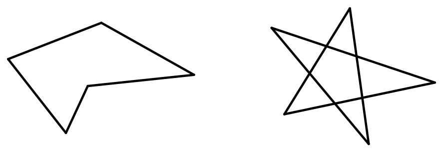
dodecagon with a hexagon attached

1.01 Polygons in the plane

A polygon is a closed figure bounded by segments called edges. A polygon may be convex or concave:

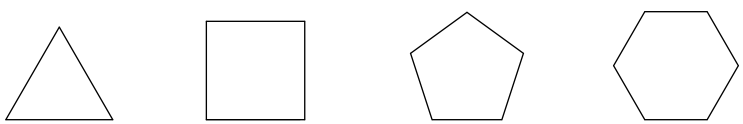


convex polygon



concave polygons

In regular polygons, the edges are of equal lengths and angles between adjacent edges are also equal.

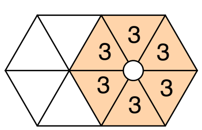


regular polygons (convex)

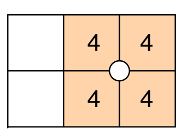
1.02 Regular tilings (tesselations) see page 5

The number of regular polygons is infinite (n ≥ 3). Three regular polygons, the triangle, the square and the hexagon, each can cover the plane without gaps or overlaps, called tilings or tesselations.

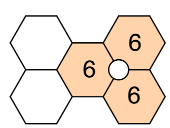
Regular (semi-regular) tilings are defined by their incidence symbols, the sequence of polygons in a vertex:



3.3.3.3.3.3



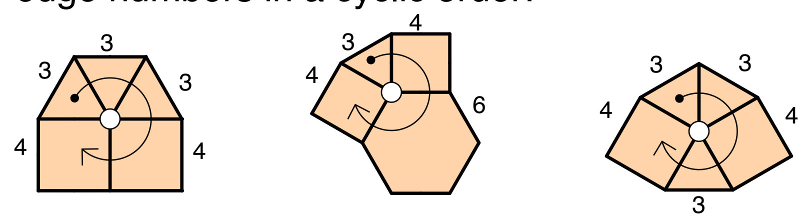
4.4.4



6.6.6

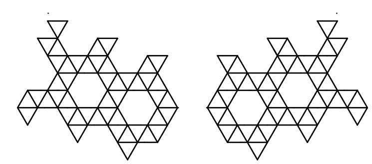
1.03 Semi-Regular tilings see page 5

If we allow more than one type of regular polygons while all the vertices are identical, we have a semi-regular tiling. There are eight variations with 3-, 4-, 6-, 8- and 12-gons. These tilings are also defined by their incidence symbols, the edge numbers in a cyclic order:



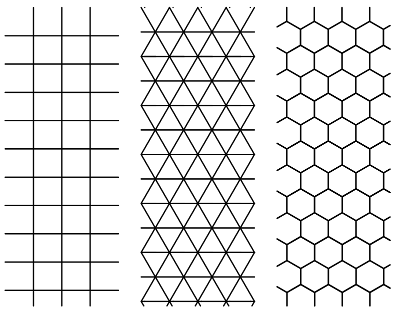
The symbol is a cyclic permutation, thus 3.3.4.3.4 is the same as 4.3.4.3.3

Regular and semi-regular tilings are symmetrical by rotations and reflections, except one: 3.3.3.3.6 has no reflection. It follows that there exists a left and a right version.

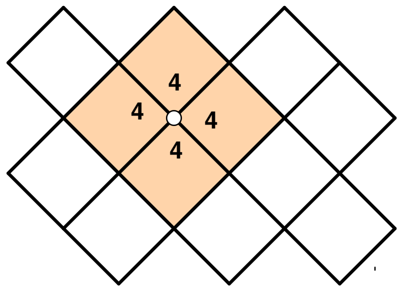


We show on page 5 in the upper-left corner the three regular tilings in a different orientation to suggest how the visual aspect of a tiling affected by selecting the x or y axis

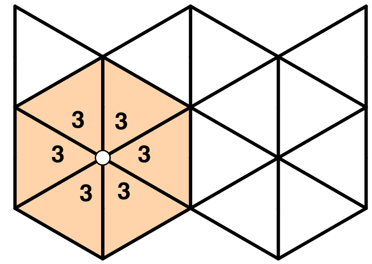
The number of different colours needed to colour the tiles such that two tiles' colours with a common edge are not the same, are either 2, 3 or 4 (see page 6).



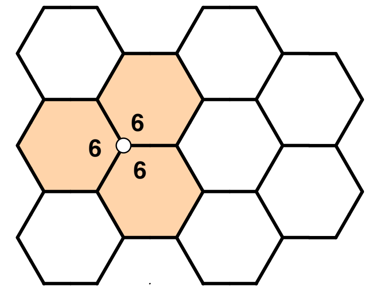
incidence symbols, plane groups →



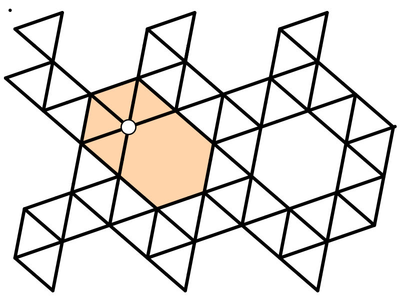
4.4.4.4 • p4m • 4^3



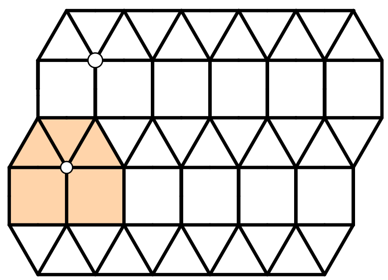
3.3.3.3.3.3 • 6pm • 3^6



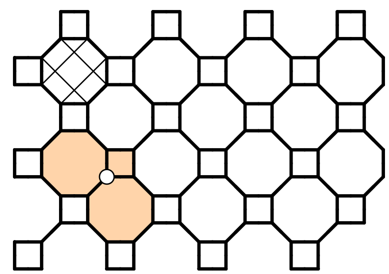
6.6.6 • p6m • 6^3



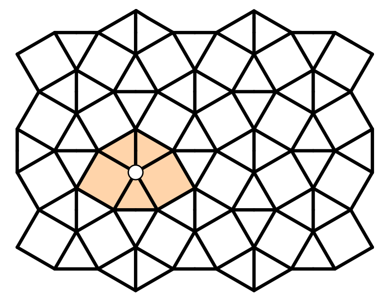
3.3.3.3.6 • p6 • $3^4.6$



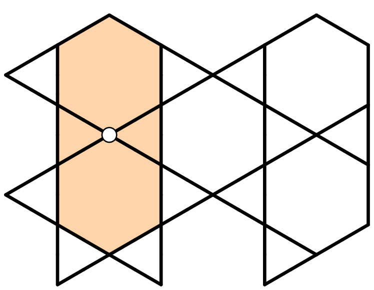
3.3.3.4.4 • cmm • $3^3.4^2$



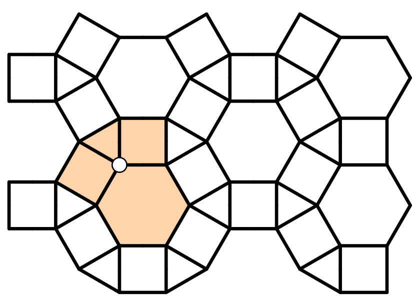
4.8.8 • p4m • 4.8^2



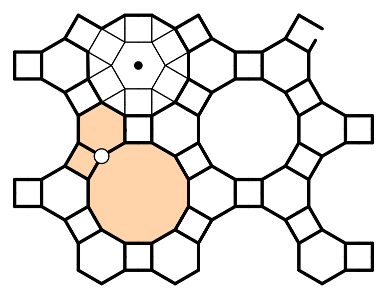
3.3.4.3.4 • p4g • $3^2.4.3.4$



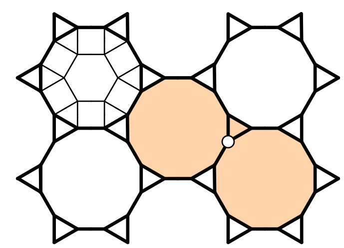
3.6.3.6 • p6m



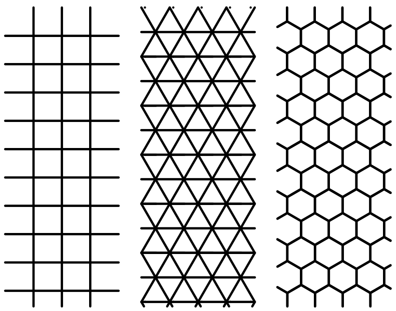
3.4.6.4 • p6m



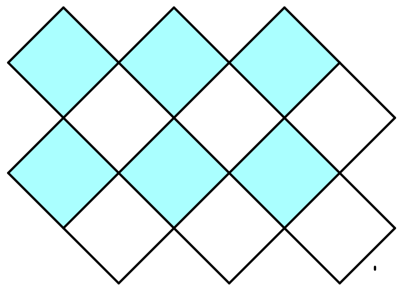
4.6.12 • 6pm



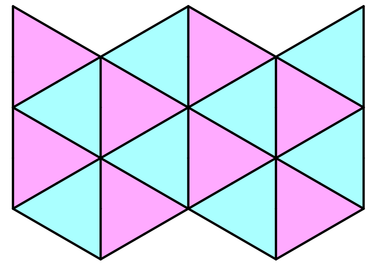
3.12.12 • p6m • 3.12^2



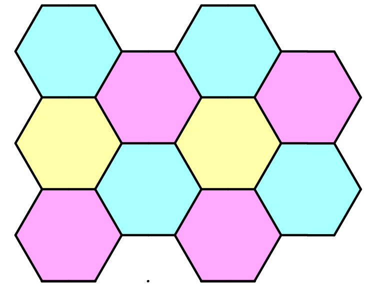
incidence symbols, plane groups →



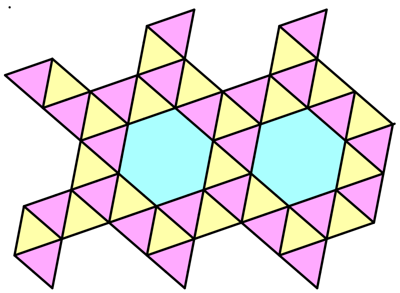
4.4.4.4 • p4m • 4^3



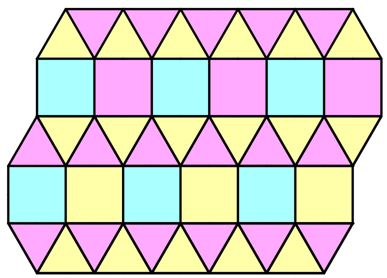
3.3.3.3.3.3 • 6pm • 3^6



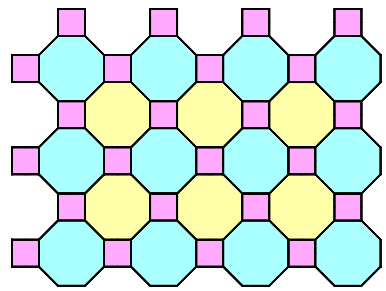
6.6.6 • p6m • 6^3



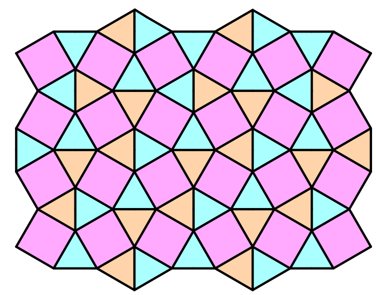
3.3.3.3.6 • p6 • $3^4.6$



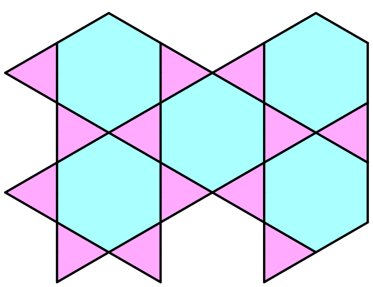
3.3.3.4.4 • cmm • $3^3.4^2$



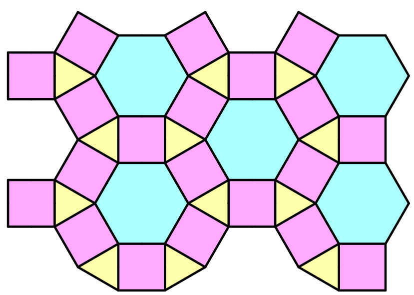
4.8.8 • p4m • 4.8^2



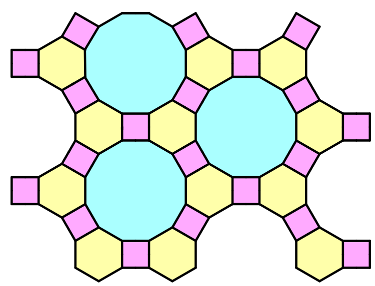
3.3.4.3.4 • p4g • $3^2.4.3.4$



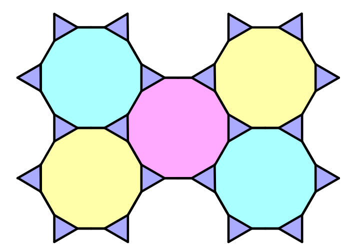
3.6.3.6 • p6m



3.4.6.4 • p6m



4.6.12 • 6pm



3.12.12 • p6m • 3.12^2

2.01 Regular polyhedral surfaces (see page 8)

This idea came accidentally during the preparation of this new version of Poly-Kit. I attended a conference on ballet in a public hall at Place des Arts in Montreal. I looked at the acoustical ceiling recently installed, and I cringed. It was a visual disarray, an insult to 3-D geometry. Next day engrossed in the possibilities, I started to think, draw, and build models, each step bringing me closer to a better understanding of this new concept: regular polyhedral surfaces (RPS).

2.02 Definitions

A regular polyhedral surface (RPH) is a connected set of infinite regular polygons with the following restrictions:

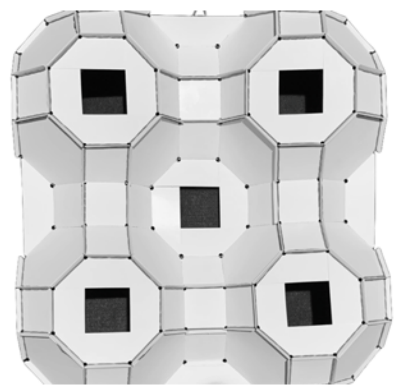
- every edge is common to two and only two faces
- two adjacent faces are never coplanar
- the arrangement of faces is ruled by one of the 17 plane symmetry groups

We introduced the concept of the incidence symbole in **1.02**. It looks like, in the case of RPH, we have one or two different incidence symbols; thus, we don't have simple instructions for the assemblage. In Exercise #2, we indicate the use of a partial regular or semi-regular polyhedron as a repetitive component of the RPH. This method was inspired by looking at the orthogonal projections of juxtaposed (space-filling) regular and semi-regular poly hedra.

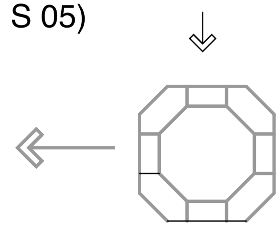
As this subject is unexplored, the question of whether the number of existing possibilities is finite or infinite remains unanswered, adding to the puzzle of the regular polyhedral surfaces.

2.03 Models

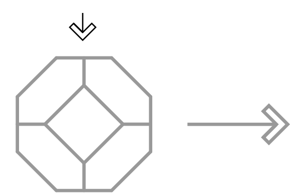
The five Exercises (see pages 16 to 20) organized for schools propose seven teams, so we devised seven alternatives. Below, we show some photos of the models made with Poly-Kit.

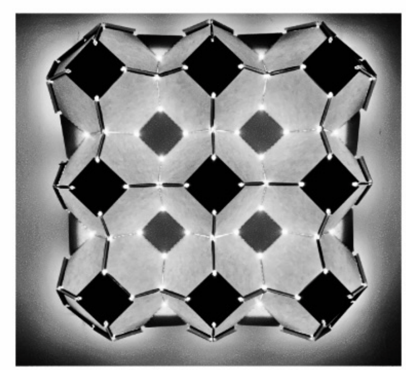


This RPH is the #6 example shown on p8. The truncated cuboctahedron is the component polyhedron (4.6.8, S 05)

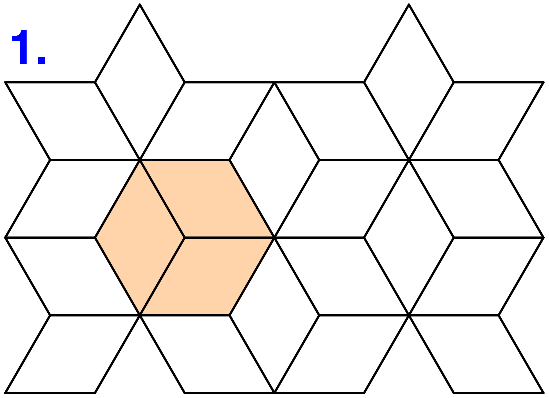


This photo is RPH made into a light fixture. The trun-truncated octahedron (4.6.6, S 03) is the component polyhedron.





1.



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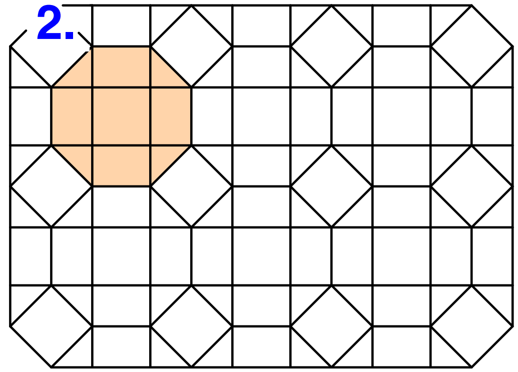

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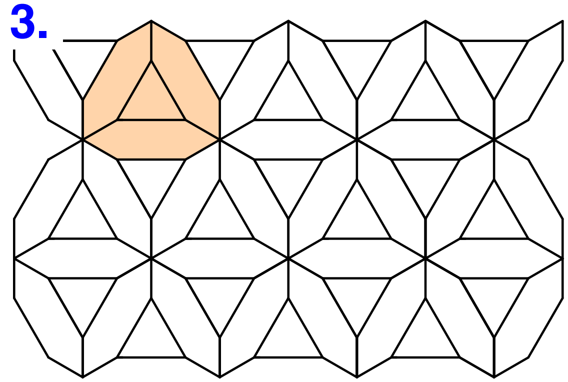

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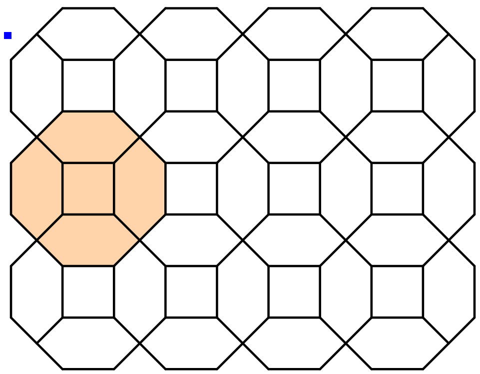
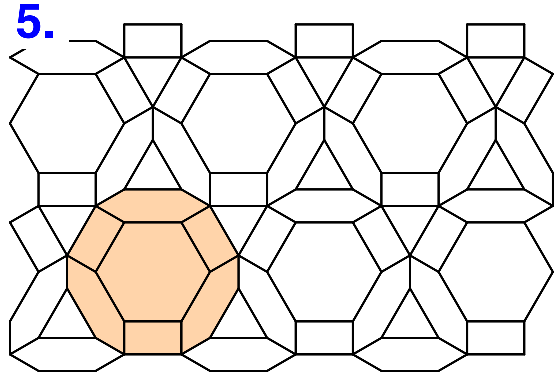
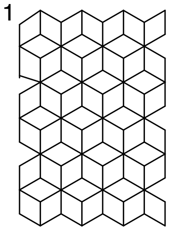


5.



Changing orientation:

1



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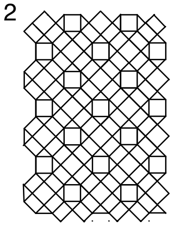

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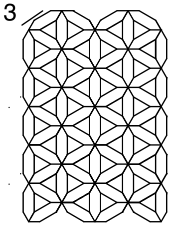
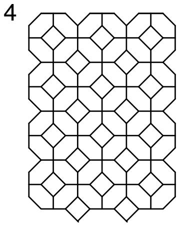
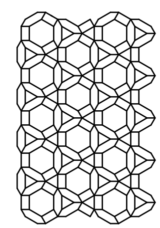


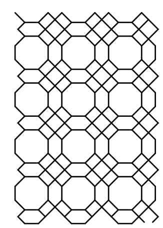
4



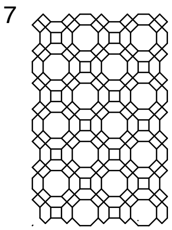
5



6

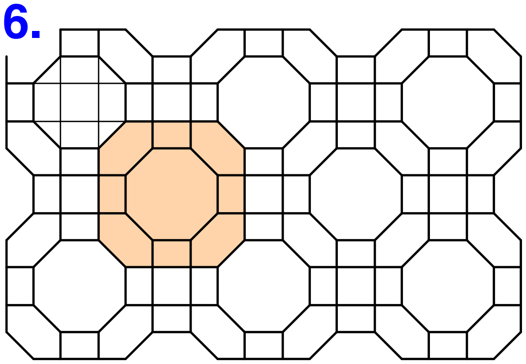


7

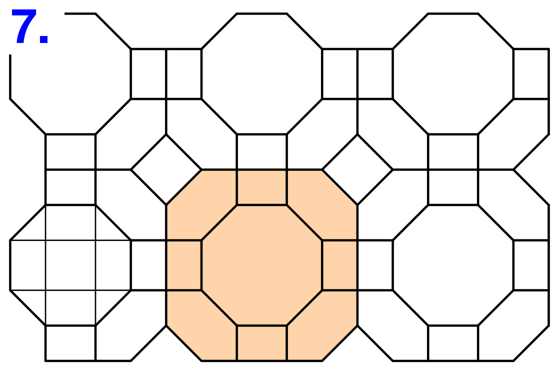


Selecting another edge to be parallel to the X-axis, it appears to be a different pattern!

6.



7.

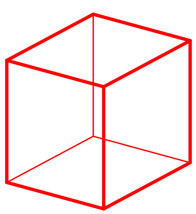


3.01 Definitions

Polyhedron: a finite connected set of polygons forming a closed surface. Every edge of each polygon belongs to just one other polygon. The polygons surrounding each vertex form a single cycle.
Convex polyhedron: also called "spherical" polyhedron, lies entierly in a half space defined by each of its face planes.
The Euler (1707-1783) formula: a remarkable relation between the vertices (V), the edges (E) and the faces (F) of **any** convex polyhedra:

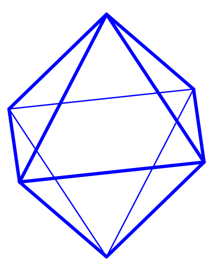
$$V - E + F = 2$$

Dual polyhedra: vertices are replaced by faces, faces are replaced by vertices, and the edges remain the same. As the term indicates, the dual of the dual polyhedron is the original polyhedron. For example, the dual of the cube is the octahedron, and the dual of the octahedron is the cube:



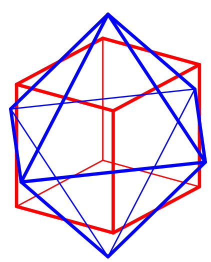
cube

8 -12 + 6 = 2



octahedron

6 -12 + 8 = 2



the dual pair

The red and the blue edges meet in their midpoints.
Regular polyhedra: all vertices and faces are regular and identical (congruent). A regular vertex is symmetrical, with the incident edges meeting at the same angle. There are five regular polyhedra, first enumerated by Platon.
Semiregular polyhedra: all faces are regular but not identical, and all vertices are identical but not regular.

	regular polyhedron	semiregular polyhedron	dual polyhdron
vertices	regular	non-tegular	regular
faces	regular	regular	non-tegular

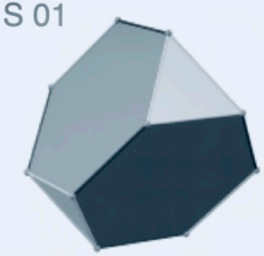
The listing of the thirteen semiregular polyhedra is credited to Archimedes.
Regular and semi-regular dual polyhedra: The duals of the five regular polyhedra are also regular polyhedra; the tetrahedron is a sef-dual, the dual of itself.
Regular prisms and antiprisms: they are also semi-regular polyhedra, they form an infinite family. Two parallel n-gons and n squares bound prisms, antiprisms are bounded by two parallel n-gons and 2n triangles.

Page 10: models of semiregular polyhedra
Page 11: Poly-kit models of 5 + 13 + 2 polyhedra
Page 12: projections of 5 + 13 + 2 polyhedra
Page 13: genealogy off 5 + 13 + 2 polyhedra
Page 14: The data on all regular, semi-regular and dual polyhedra are tabulated

Use the incidence symbols described on page 4 and listed in the Table (page 14) for the assemblage of polyhedra.

The 13 + 2 semi-regular polyhedra

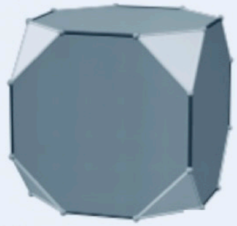
S 01



3.6.6

truncated tetrahedron

S 02



3.8.8

truncated cube

S 03

4.6.6

truncated octahedron

S 04

3.4.3.4

Cuboctahedron

S 05

4.6.8

truncated cuboctahedron

S 06

3.4.4.4

Rhombicuboctahedron

S 07

3.3.3.3.4

snub cube

S 08

3.10.10

truncated dodecahedron

S 09

5.6.6

truncated icosahedron

S 10

3.5.3.5

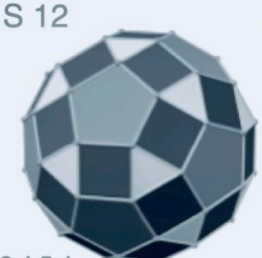
icosidodecahedron

S 11

4.6.10

truncated icosidodecahedron

S 12



3.4.5.4

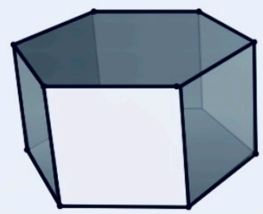
rhombicosidodecahedron

S 13

3.3.3.3.5

snub dodecahedron

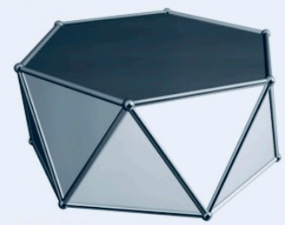
S 14



4.4.6

hexagonal prisms

S 15



3.3.3.6

hexagonal antiprisms

The 5 regular polyhedra and their duals

R 01

3.6.6.

R 02

3.3.3.3

R 03

4.4.4

R 04

3.3.3.3.3.

R 05

5.5.5

The 13 + 2 semi-regular polyhedra

S 01

3.6.6

S 02

3.8.8

S 03

4.6.6

S 04

3.4.3.4

S 05

4.6.8

S 06

3.4.4.4

S 07

3.3.3.3.4

S 08

3.10.10

S 09

5.6.6

S 10

3.5.3.5

S 11

4.6.10

S 12

3.4.5.4

S 13

3.3.3.3.5

Dual polyhedra

D04

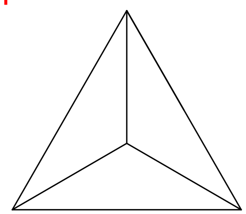
3.4.3.4

D 10

3.5.3.5

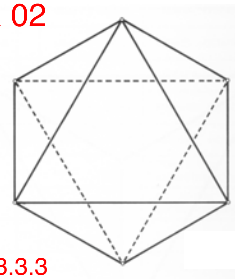
The 5 regular polyhedra

R 01



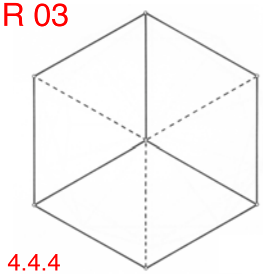
3.6.6.

R 02



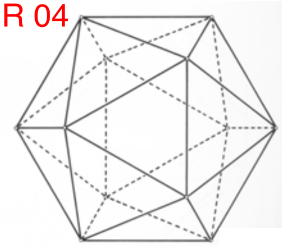
3.3.3.3

R 03



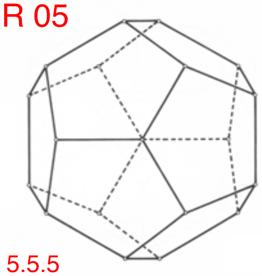
4.4.4

R 04



3.3.3.3.3.

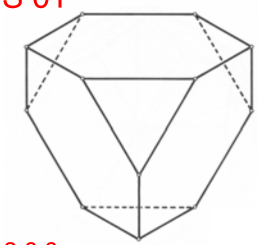
R 05



5.5.5

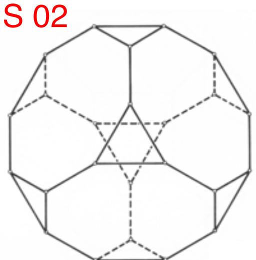
The 13 + 2 semi-regular polyhedra

S 01



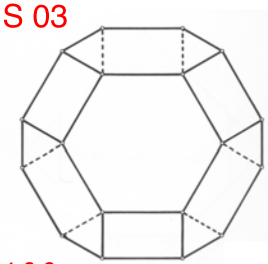
3.6.6

S 02



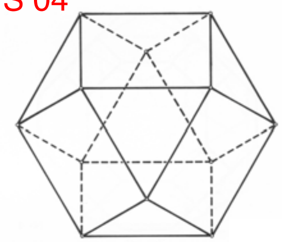
3.8.8

S 03



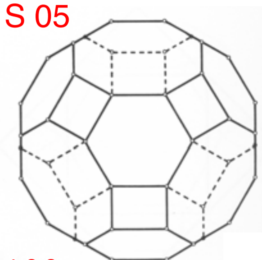
4.6.6

S 04



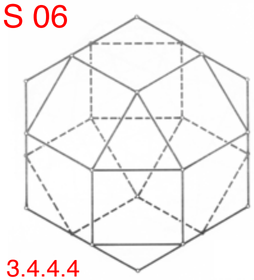
3.4.3.4

S 05



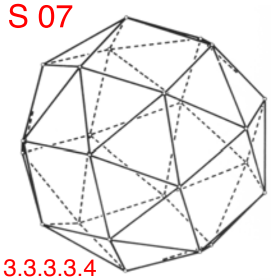
4.6.8

S 06



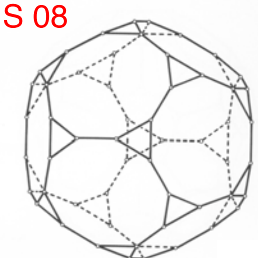
3.4.4.4

S 07



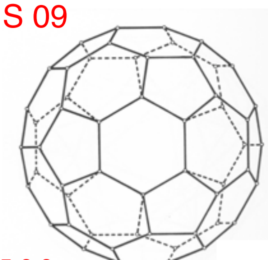
3.3.3.3.4

S 08



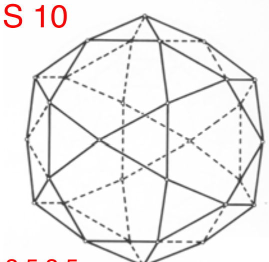
3.10.10

S 09



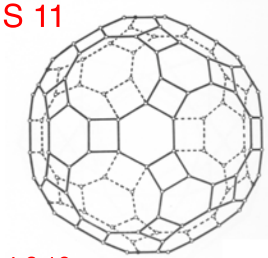
5.6.6

S 10



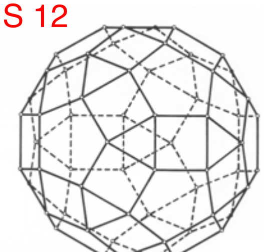
3.5.3.5

S 11



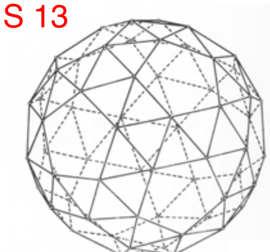
4.6.10

S 12



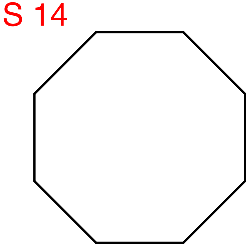
3.4.5.4

S 13



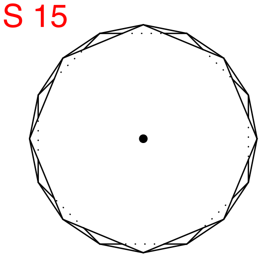
3.3.3.3.5

S 14



n.4.4

S 15



n3.3.3

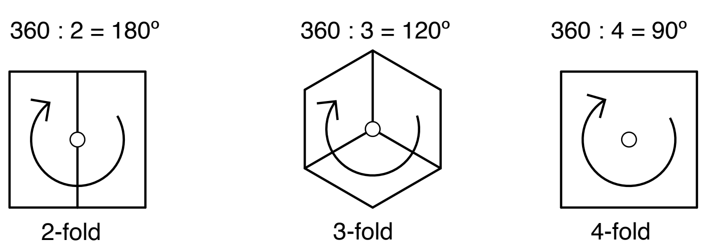
3.02 Genealogy

Here, we briefly explore nature's inherent order and beauty as it applies to this family of polyhedra. The tetrahedron is the simplest polyhedron with four faces. All other polyhedra may be derived from it by successive truncations. The illustration on the right depicts the dual relationship among the four other regular polyhedra placed in two columns.

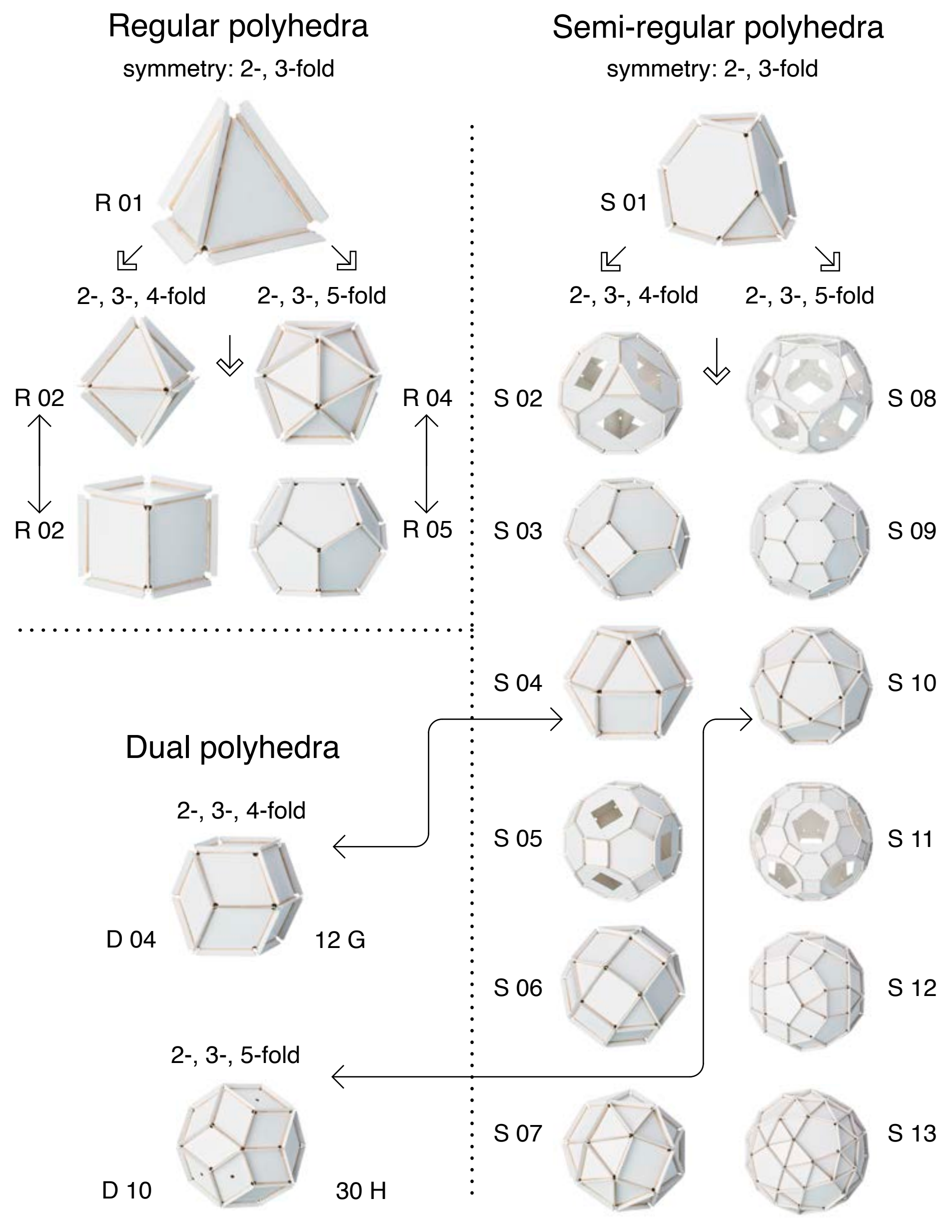
The truncated tetrahedron heads the two columns of the semi-regular polyhedra. The tongue-twisting Greek names suggest their interrelations.

3.03 Symmetry

We mentioned symmetry in **1.03,** here, we have to elaborate this concept to explain the two columns of polyhedra on the right. The rotational symmetry of n-fold means that the polyhedron possesses an axe such that when rotated with an angle of 360/n, the rotated figure coincides with the original one. Here, we show the rotational symmetries of the cube, 2-,3-,4 fold:



This illustration explains why the two columns on the right.



3.04 The 5 regular polyhedra

code	name	symbol	faces	v	e	f
R 01	tetrahedron	3.3.3	4 A	4	6	4
R 02	octahedron	3.3.3.3	8 A	6	12	8
R 03	cube	4.4.4.	6B	8	12	6
R 04	icosahedron	3.3.3.3.3	20A	12	30	20
R 05	dodecahedron	5.5.5	12 C	20	30	12

3.05 The 13 + 2 semi-regular polyhedra

code	name	symbol	faces	v	e	f
S 01	truncated tetrahedron	3.6.6	4A+4D	12	18	8
S 02	truncated cube	3.8.8	8A+24E	24	36	14
S 03	truncated octahedron	4.6.6	6B+8D	24	36	14
S 04	Cub-octahedron	3.4.3.4	8A+6B	12	24	14
S 05	truncated cuboctahedron	4.6.8	12B+8D+24E	48	72	26
S 06	Rhombi-cuboctahedron	3.4.4.4	8A+18B	24	48	26
S 07	snub-cube	3.3.3.3.4	32A+6B	24	60	38
S 08	truncated dodecahedron	3.10.10	20A+60F	60	90	32
S 09	truncated icosahedron	5.6.6	12C+20D	60	90	32
S 10	icosi-dodecahedron	3.5.3.5	20+12C	30	60	32
S 11	truncated icosidodecahedron	4.6.10	30B+20D+60F	120	180	62
S 12	rhomb-icosidodecahedron	3.4.5.4	20A+30B+12C	60	120	62
S 13	snub-dodecahedron	3.3.3.3.5	80A+12C	60	150	92
S 14	n-gonal-prisms	n.4.4	nB+2n	2n	3n	n+2
S 15	n-gona-antiprisms	n.3.3.3	2nA+2n	2n	n	2n+2

3.06 The 5 dual regular polyhedra

code	name	symbol	faces	f	e	v
R 01	tetrahedron	3.3.3	4 A	4	6	4
R 03	cube	4.4.4.	6B	8	12	6
R 02	octahedron	3.3.3.3	8A	6	12	8
R 05	dodecahedron	5.5.5	12 C	20	30	12
R 04	icosahedron	3.3.3.3.3	20 A	12	30	20

3..07 The 13 + 2 dual semi-regular polyhedra

code	name	symbol	faces	f	e	v
D 01	triakis tetrahedron	3.6.6	-	12	18	8
D 02	triakis octahedron	3.8.8	-	24	36	14
D 03	tetrakis hexahedron	4.6.6	-	24	36	14
D 04	rhombic dodecahedron	3.4.3.4	12G	12	24	14
D 05	hexakis octahedron	4.6.8	-	48	72	26
D 06	trapezoidal icositetrahedron	3.4.4.4	-	24	48	26
D 07	triakis icositetrahedron	3.3.3.3.4	-	24	60	38
D 08	triakis icosahedron	3.10.10	-	60	90	32
D 09	pentakis dodecahedron	5.6.6	-	60	90	32
D 10	rhombic triacontahedron	3.5.3.5	30H	30	60	32
D 11	hexakis icosahedron	4.6.10	-	120	180	62
D 12	trapezoidal hexacontahedron	3.4.5.4	-	60	120	62
D 13	pentagonal hexacontahedron	3.3.3.3.5	-	60	150	92
D 14	n-gonal dipyramids	n.4.4	-	2n	3n	n+2
D 15	n-gonal trapezohedra	n.3.3.3	-	2n	n	2n+2

The following notes are addressed to the teachers at any level for the organization of classroom activities with Poly-Kits. The concepts of the five exercises (pages 16 to 20) are based on many years of classroom experience with the first version of Poly-Kit.

The classroom activity is based on dividing the class into seven teams, assuming the average number of students to be around twenty worldwide.

The sequence of the five exercises starts with 2-D (#1), then 2 !/2-D (? #2) and continues with three 3-D exercises. This sequence does not correspond to the degree of difficulties or the amount of time required to complete a session. Exercise #1 is time-consuming but not difficult, exercise #2 is time-consuming and difficult. Exercises #3, @4 and #5 are in the correct sequence of difficulty. We suggest 1 or 2 hours of classroom time per exercise.

We suggest the teachers read the manual and build them-self at least one model for each exercise. Preparation for the class should include copying certain pages of this manual for distribution and selecting the pertinent pages (with blue title blocks) for slide projections.

We made a reference to the type of language and the depth of explanation we use in this manual on page 2. Here, we want to expand on this aspect. We believe that teachers should acquire more knowledge than the content they present in class; the easy access to the internet (i.e. Wikipedia) allows a further exploration of this wonderful universe of polyhedra and 3-D geometry in general.

Please note, that exercises #1 and @2 required two complete set of Poly-Kit's.

We suggest the following steps in class:

- Introduction to Poly-Kit, slide of the cover page
- the content of Poly-Kit, page 3
- the three products ot Poly-Kit, pages 5, 8, 11
- explain the incidence symbol on a model
- distribution of copies of pertinent pages
- popping out the polygons from the sheets (students)
- distribution of the polygons (students)

For the incidence symbols, see the table on page 14

code	name	symbol	faces	v	e	f
S 01	truncated tetrahedron	3.6.6	4A+4D	12	18	8

The symbol 3.3.6 means that at every vertex, two triangles and one hexagon meet. If you follow this simple rule at every vertex, the polyhedron will be assembled by "itself."

We left it to the last our most important recommendation: for the best results, allow the students a brief time after an exercise to freely create new forms by combining polygons into never-seen-before spatial models.

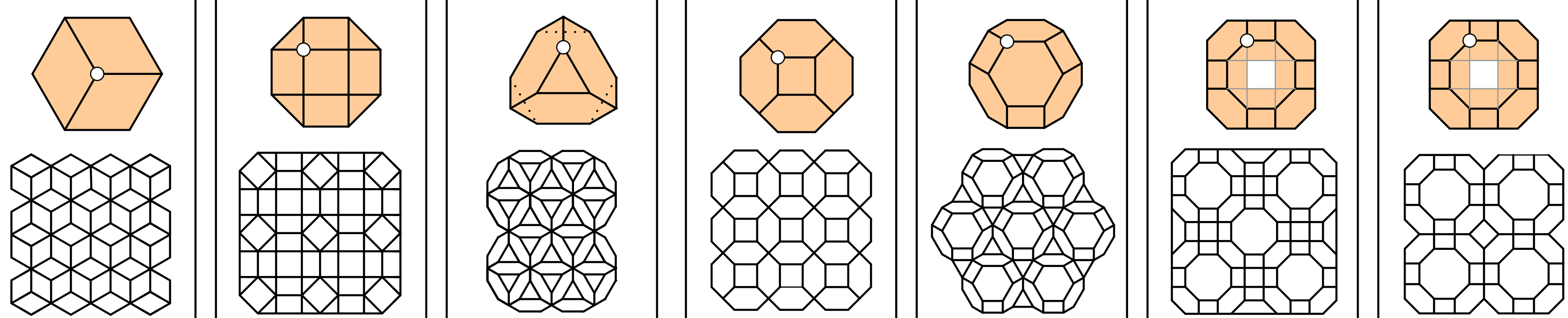
You will be surprised by the enthusiasm to create new forms and by the richness of ideas

We hope you will enjoy Pol-Kit!

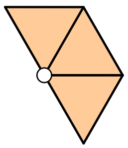
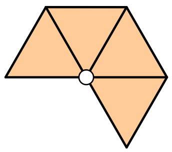
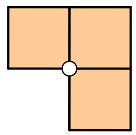
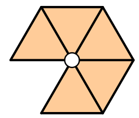
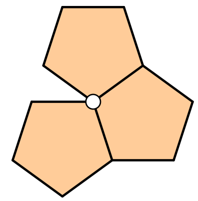
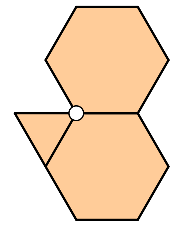
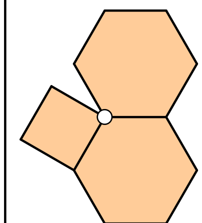
• Exercise #1: 3 Regular, 8 Semiregular Tilings •

Team 1.	Team 2.	Team 3.	Team 4.	Team 5.	Team 6.	Team 7.
3.3.3.3.3.3	4.4.4.	6.6.6	3.3.3.4.4	4.8.8	4.6.12	3.12.12
3.3.4.3.4	3.3.3.3.6	3.6.3.6	3.4.6.4			
86 faces	59 faces	28 faces	88 faces	76 faces	67 faces	80 faces
62	45	13	52		18	50
24	12		31	20	38	30
	2	15	5		11	
				56		

	Team 1.	Team 2.	Team 3.	Team 4.	Team 5.	Team 6.	Team 7.
	4.4.4	4.4.4+3.4.3.4 3.4.4.4	3.6.6	4.6.6	3.6.6+4.6.6+ 4.6.8.	4.4.4+4.6.6+ 4.6.8	4.4.4+4.6.6+ 4.6.8



	Team 1.	Team 2.	Team 3.	Team 4.	Team 5.	Team 6.	Team 7.
	50 faces	69 faces	56 faces	33 faces	61 faces	72 faces	57 faces
		32	20		12		
	50	37		9	21	28	17
			36	24	28	16	16
						28	24

Team	Vertex	Vertex figure	Faces	Triangles	Squares	Pentagons	Hexagons
Team 1.	3.3.3		4 faces	4	6		
Team 2.	3.3.3.3		8 faces	8			
Team 3.	4.4.4.		6 faces		6		
Team 4.	3.3.3.3.3		20 faces	20			2
Team 5.	5.5.5		12 faces			12	
Team 6.	3.6.6		8 faces	4			4
Team 7.	4.6.6		14 faces		6		8

• Exercise #4: Regular and Semi-regular Polyhedra 2 •

	Team 1.	Team 2.	Team 3.	Team 4.	Team 5.	Team 6.	Team 7.
Vertex type	4.4.4.	3.3.3	3.6.6	3.3.3.3.3	4.6.6	5.5.5	3.3.3.3
Vertex type	5.6.6	3.5.3.5	3.4.4.4	4.4.6	3.4.3.4	3.3.3.6	3.3.3.5
Faces	38 faces	36 faces	34 faces	28 faces	28 faces	26 faces	20 faces
Triangles		24	12	20	8	12	18
Squares	6		18	6	12	6	6
Pentagons	12	12				12	2
Hexagons	20		4	2	8	2	

Team 1.	Team 2.	Team 3.	Team 4.	Team 5.	Team 6.	Team 7.
3.8.8	4.6.8	3.10.10	4.6.10	3.3.3.3.4	3.4.5.4	3.3.3.3.5
14 faces	26 faces	32 faces	62 faces	38 faces	62 faces	92 faces
8		20		32	20	80
	12		30	6	30	
					12	12
	8		20			
24	24					
		60	60			